Math B - Unit 5 Study Guide: Fractions

BE SURE TO WATCH ALL VIDEOS AND COMPLETE ALL EXAMPLES WITHIN EACH OLS LESSON

Lesson 1: Equivalent Fractions

Every fraction can be written as a decimal and every decimal can be written as a fraction. As a result, you could do just about all math with only fractions or only decimals, but decimals are used for certain applications just as fractions are used for others. For example, carpenters use fractions and mixed numbers quite a bit. Anybody building a house or a deck deals with lots of fractions.

VOCABULARY

Fraction: a number in the form of $\frac{a}{b}$ where a and b are integers and b ≠ 0

Denominator: the part of the fraction below the bar

Numerator: the part of the fraction above the bar

Simplest or lowest terms of a fraction: when the numerator and denominator have no common factor other than 1

Equivalent Fractions: Fractions that have the same value

Steps for putting a fraction in lowest or simplest terms:

1. Find the factors of the numerator and denominator
2. The **Greatest Common Factor** is the largest number in both lists *(It will be equal to or smaller than your original number)*
3. ***Divide*** the numerator and denominator by any common factor, preferably the greatest common factor (GCF)

Example: $\frac{8}{24}$

Factors of 8: 1, 2, 4, 8

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

GCF = 8

Simplify: $\frac{8}{24} \div \frac{8}{8} = \frac{1}{3}$

OFFLINE WORK

- Read pages 143–145.
- Complete Problems 1–23 odd on pages 145–146.
- Complete Problems 2–24 even on pages 145–146 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*

Determine if Fractions are Equivalent:

1. Write each fraction in lowest terms
2. Check to see if they are equivalent

Example: $\frac{3}{18}\ and\ \frac{6}{24}$

1. $\frac{3}{18} \div \frac{3}{3} = \frac{1}{6}$

   $\frac{6}{24} \div \frac{6}{6} = \frac{1}{4}$
2. $\frac{1}{4} \neq \frac{1}{6}$

Finding Equivalent Fractions:

HOT TIP! You can multiply OR divide the numerator and denominator of a fraction by the same number to get an equivalent fraction

Example: $\frac{3}{4} \cdot \frac{5}{5} = \frac{15}{20}$ Multiply numerator and denominator by 5

$\frac{3}{4}\ and\ \frac{15}{20}$ are equivalent fractions

Example: $\frac{16}{22} \div \frac{2}{2} = \frac{8}{11}$ Divide numerator and denominator by 2

$\frac{16}{22}\ and\ \frac{8}{11}$ are equivalent fractions

Using Equivalent Fractions to Solve Equations:

Example:

$\frac{1}{2} = \frac{?}{8}$

1. Denominator changed from 2 to 8: what do you multiply by 2 to get 8? 4
2. Multiply the numerator by the same number (4 x 1 = 4)
3. $\frac{1}{2} = \frac{4}{8}$

Lesson 2: Multiply Fractions

Multiply Fractions:

Multiply the numerator and the denominator

(Top times top, bottom times bottom)

Simplify

****You don't need a common denominator to multiply fractions – YAYYYYY**

Lesson 3: Divide Fractions

Reciprocal: 2 numbers whose product is 1. Turn the fraction upside down. In other words, swap the numerator and denominator. **Just Flip It!**

Examples:

Fraction	Reciprocal
$\frac{3}{8}$	$\frac{8}{3}$
$\frac{-2}{3}$	$\frac{-3}{2}$
$6 = \frac{6}{1}$	$\frac{1}{6}$

HOT TIP! To change a whole number (like 6 or -17) to a fraction, give it a denominator of 1

Dividing Fractions:

Flip the second fraction upside down

Multiply the new flipped fraction times the first fraction

Dividing Fractions

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, where b, c, and d are not 0.

Example:

$\frac{1}{5} \div \frac{3}{10} = \frac{1}{5} \cdot \frac{10}{3}$	Multiply by the reciprocal.
$= \frac{10}{15}$	Multiply the numerators and denominators.
$= \frac{2}{3}$	Write in lowest terms.

VIDEO: Dividing Fractions

http://k12.http.internapcdn.net/k12_vitalstream_com/CURRICULUM/271165/CURRENT_RELEASE/MS_PA_S1_04_04_WHB_divide_fractions.htm

OFFLINE WORK

- Read pages 151–153.
- Complete Problems 1–4, 9–15, and 35–36 on pages 153–154.
- Complete Problems 5–6 and 17–21 odd on pages 153–154 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Lesson 4: CORE FOCUS: Rational Numbers

Suppose the price of a jacket is $36. The jacket is on sale for 25% off. To determine the sale price of the jacket, you can multiply $36 by a rational number.

In this lesson, you'll solve real-world problems involving rational numbers. In many of these problems, you'll use rational numbers to model increases and decreases.

VIDEO: Core Focus: Rational Numbers

OFFLINE WORK:

- Read pages 155–157.
- Complete Problems 1–3 on page 157.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Lesson 5: SKIP (optional)

Lesson 6: Common Denominators

You've learned to determine if two fractions are equivalent fractions. Knowing how to write equivalent fractions will help you express fractions using the same denominator. Finding the ***least common denominator (****LCD) of a set of fractions is a skill you will use repeatedly. In this lesson, you learn how to find the LCD.*

Multiple: of a number is the product of that number and a counting number

Example: Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24…
Multiples of 9: 9, 18, 27, 36, 45, 54, 63, 72…

Least Common Multiple (LCM): of 2 or more numbers is the least multiple that is common to all the numbers

Least Common Denominator (LCD): of 2 or more fractions is the LCM of their denominators

Steps to find the Least Common Denominator (LCD) of 2 Fractions:

1. List the multiples of each denominator
2. Circle the lowest number in both lists

Example: $\frac{1}{3}$ List the multiples of 3: 3, 6, 9, 12, 15, 18, 21, …
$\frac{1}{6}$ List the multiples 6: 6, 12, 18, 24, …

6 is the lowest number in each list, so it is the LCD of these 2 fractions

HOT TIP! To find any common denominator (not necessarily the least common denominator): multiply the 2 denominators together

Compare Fractions:

1. Convert all fractions to the same denominator
2. Compare the numerators

Example: Which is larger: $^{3}/_{8}$ or $^{5}/_{12}$?

If you multiply 8 × 3 you get 24 , and if you multiply 12 × 2 you also get 24, so let's try that (*important: what you do to the bottom, you must also do to the top*):

$$\frac{3}{8} \overset{\times 3}{=} \frac{9}{24} \quad \text{and} \quad \frac{5}{12} \overset{\times 2}{=} \frac{10}{24}$$

It is now easy to see that $^{9}/_{24}$ is smaller than $^{10}/_{24}$, (because 9 is smaller than 10).

→ so $^{5}/_{12}$ is the larger fraction.

Finding the LCD

A common denominator is a common multiple of two or more denominators. The least common denominator (LCD) is the least of all the common multiples.

You can find the LCD of two or more fractions by finding the LCM of their denominators.

Find the least common denominator (LCD) of $\frac{3}{10}$ and $\frac{4}{15}$.

List the multiples of the first denominator (10).

multiples of 10: 10, 20, 30, 40, 50, 60, ...

List the multiples of the second denominator (15).

multiples of 15: 15, 30, 45, 60, 75, ...

Find the least of the common multiples.

multiples of 10: 10, 20, 30, 40, 50, 60, ...
multiples of 15: 15, 30, 45, 60, 75, ...
The LCD is 30.

Equivalent Fractions

You can rename a fraction as an equivalent fraction by multiplying the numerator and denominator by the same number.

When you multiply the numerator and denominator by the same number, you are really multiplying the fraction by 1.

Express $\frac{3}{10}$ and $\frac{4}{15}$ using the same denominator.

Find the LCD.

The LCD is 30.

Rename each fraction with the LCD.

$\frac{3}{10} = \frac{3}{10} \cdot \frac{3}{3}$	$\frac{4}{15} = \frac{4}{15} \cdot \frac{2}{2}$	Multiply each fraction by 1.
$= \frac{9}{30}$	$= \frac{8}{30}$	Simplify.
$\frac{3}{10}$ becomes $\frac{9}{30}$	$\frac{4}{15}$ becomes $\frac{8}{30}$	

VIDEO: Comparing Fractions with unlike Denominators
http://k12.http.internapcdn.net/k12_vitalstream_com/CURRICULUM/318080/CURRENT_RELEASE/MS_PA_S1_04_07_WHB_comparing_fractions_unlike_denominators.htm

OFFLINE WORK

- Read pages 158–161.
- Complete Problems 1–31 odd on pages 161–162.
- Complete Problems 2–32 even on pages 161–162 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Lesson 7 & 8: Add and Subtract Fractions

You may have heard the expression, "It's like comparing apples and oranges." The expression means that you can't compare two things that are essentially different and expect your comparison to make any sense. Similarly, you can't add or subtract two fractions whose denominators are different. It would be like comparing apples and oranges. In this lesson, you will learn how to add and subtract fractions

Add and Subtract Fractions with the Same Denominator:

1. Add or subtract the numerator
2. Keep the same denominator

Example: $\frac{2}{6} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6}$

Add and Subtract Fractions with a Different Denominator:

When you add or subtract fractions with different denominators, you must find equivalent fractions with the same denominator before you can add or subtract

Example:

$\frac{3}{5} - \frac{2}{7}$ Find a common denominator and create equivalent fractions

Select the denominator of the second fraction (7) and multiply the top and bottom of the first fraction (3/5) by that number

$\frac{7}{7} \times \frac{3}{5} = \frac{21}{35}$

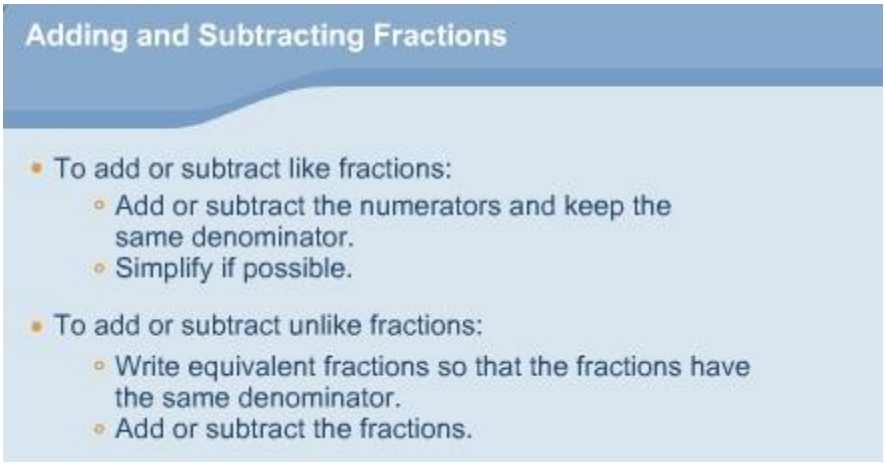
Select the denominator of the first fraction (5) and multiply the top and bottom of the second fraction (2/7) by that number

$\frac{5}{5} \times \frac{2}{7} = \frac{10}{35}$

These two fractions (21/35 and 10/35) have common denominators -- the number 35 on the bottom of the fraction.

We can now subtract, because the two new fractions have a common denominator:

$\frac{21}{35} - \frac{10}{35} = \frac{11}{35}$



OFFLINE WORK

- Read pages 163–165.
- Complete Problems 1–19 odd on page 166.
- Complete Problems 2–20 even on page 166 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

VIDEO: Solve a Word Problem
http://k12.http.internapcdn.net/k12_vitalstream_com/CURRICULUM/271166/CURRENT_RELEASE/MS_PA_S1_04_09_WHB_word_problem.htm

More OFFLINE WORK

- Read pages 163–165.
- Complete Problems 42–44 on page 167.
- Complete Problem 45–46 on page 167 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Lesson 9 – SKIP – Optional

Lesson 10: Working with Mixed Numbers

Sometimes amounts of things are greater than 1, so not all fractions are proper fractions. You might need 2½ cups of sugar to bake a cake or 4¼ gal of gas to fill a gas tank. In this lesson, you will learn to add and subtract mixed numbers and improper fractions.

Mixed Number: a number made up of a whole number and a proper fraction (a fraction less than 1 with a numerator less than the denominator)

Examples: 4 ½ , 7 ¼

Improper Fraction: a fraction greater than or equal to 1. The numerator is greater than or equal to the denominator

Examples: $\frac{11}{4}, \frac{9}{7}$

Proper Fraction: a fraction less than 1. The numerator is less than the denominator

Examples: $\frac{2}{3}, \frac{4}{9}$

Convert an Improper Fraction to a Mixed Number:

1. Divide the numerator by the denominator to find the quotient and the remainder
2. The quotient is the whole number, the remainder is the numerator, the denominator remains the same

Example: Convert 11/4 to a mixed fraction.

Divide:

→ 11 ÷ 4 = 2 with a remainder of 3

Write down the 2 and then write down the remainder (3) above the denominator (4), like this:

$$2\frac{3}{4}$$

Convert a Mixed Number to an Improper Fraction:

1. Multiply the denominator times the whole number
2. Add the numerator to the number from step 1 – this is the new numerator
3. Keep the same denominator

Example: Convert 3 $^{2}/_{5}$ to an improper fraction.

Multiply the whole number by the denominator:

→ 3 × 5 = 15

Add the numerator to that:

→ 15 + 2 = 17

Then write that down above the denominator, like this:

$$\frac{17}{5}$$

Adding Mixed Numbers: Method 1:

Add the whole numbers
Add the fractions (must have common denominators)
Simplify

Adding Mixed Numbers: Method 2

Convert to improper fractions
Add the fractions (must have common denominators)
Simplify (convert back to mixed numbers)

VIDEO: Add Mixed Numbers
http://k12.http.internapcdn.net/k12_vitalstream_com/CURRICULUM/271167/CURRENT_RELEASE/MS_PA_S1_04_10_WHB_add_mixed_numbers.htm

OFFLINE WORK:

- Read pages 168–171.
- Complete Problems 1–25 odd and 33–35 odd on pages 171–172.
- Complete Problems 2–26 even and 32–34 even on pages 171–172 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

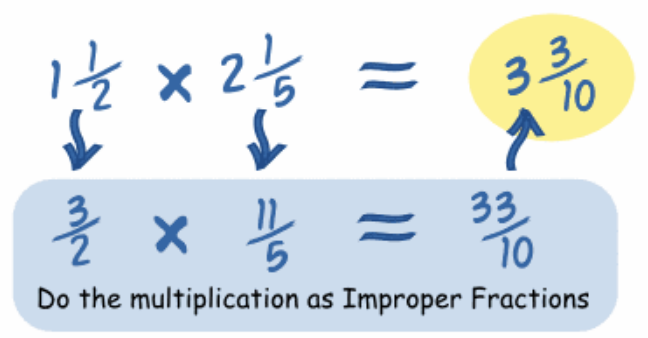
Lesson 11: Multiplying and Dividing Mixed Numbers

Think of some real-world examples of mixed numbers. Maybe you walked 5½ miles this week, used a recipe that called for 1¾ cups of kidney beans, or watched a movie that was 2¼ hours long. There are times when you need to perform multiplication and division using mixed numbers. You might want to find your average walking distance per day or the amount of kidney beans to use if you make 1½ times the recipe. In this lesson, you will learn how to multiply and divide mixed number.

Multiply Mixed Numbers: Convert each mixed number to an improper fraction and multiply
Example:

What is 1 ½ x 2 ⅕ ?

If you know how to go from Mixed Fraction to Improper Fractions and back again it is easy ...



Divide Mixed Numbers:

1. Convert the mixed numbers to improper fractions
2. Flip the second fraction and multiply

Reciprocal of a Mixed Number

Example: What is the reciprocal of $2\ ^1/_3$?

1. Convert it to an improper fraction: $2\ ^1/_3 = {}^7/_3$
2. Turn it upside down: $^3/_7$

The Answer is: $^3/_7$

Multiplying Mixed Numbers

1: Convert the mixed numbers to improper fractions.
2: Multiply the numerators and multiply the denominators.
3: Write the product in lowest terms.

Dividing Mixed Numbers

To divide two mixed numbers:
1) Convert any mixed numbers to improper fractions.
2) Multiply by the reciprocal.
3) Carry out the multiplication, then simplify.

Example:

$$8\frac{3}{4} \div \frac{5}{6} = \frac{35}{4} \div \frac{5}{6}$$
$$= \frac{35}{4} \cdot \frac{6}{5}$$
$$= \frac{7}{2} \cdot \frac{3}{1}$$
$$= \frac{21}{2}$$
$$= 10\frac{1}{2}$$

VIDEO: Dividing Mixed Numbers

http://k12.http.internapcdn.net/k12_vitalstream_com/CURRICULUM/318081/CURRENT_RELEASE/MS_PA_S1_04_12_WHB_dividing_mixed_numbers.htm

OFFLINE WORK

- Read pages 173–175.
- Complete Problems 1–33 odd on page 176.
- Complete Problems 2–26 even and 30–32 even on page 176 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Lesson 12: Equations with Fractions

Once you know how to add, subtract, multiply, and divide fractions, you're ready to solve equations that involve fractions. In this lesson, you will learn to do just that.

Remember:

1. Isolate the variable
2. If you are adding on the same side as the variable, subtract that same number from both sides of the equal sign
3. If you are subtracting on the same side as the variable, add that same number to both sides of the equal sign
4. If you are multiplying on the same side as the variable, divide that same number on both sides of the equal sign; you can also multiply both sides by the reciprocal
5. If you are dividing on the same side as the variable, multiply that same number on both sides of the equal sign

Examples:

$\frac{y}{3} = 7$ — y is being divided by 3

multiply both sides of the equation by 3

$(3)\frac{y}{3} = 7(3)$ — solve for y

$y = 21$

m + ½ = 7 — ½ is being added to m

m + ½ - ½ = 7 – ½ — subtract ½ from both sides of the equation

$m = \frac{14}{2} - \frac{1}{2}$ — convert 7 to 14/2 and subtract

$m = \frac{13}{2} = 6\frac{1}{2}$ — solve for m

$\frac{-3}{5}a = 27$ — a is being multiplied by -3/5

$\left(-\frac{5}{3}\right)\frac{-3}{5}a = 27\left(-\frac{5}{3}\right)$ — multiply both sides of the equation but the reciprocal: -5/3

$a = -\frac{27(5)}{3} = -45$ — solve for a

OFFLINE WORK

- Read pages 177–179.
- Complete Problems 1–27 odd on page 180.
- Complete Problems 2–28 even on page 180 for extra practice (optional).

- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Lesson 13: CORE FOCUS: Fractions and Mixed Numbers

Suppose you have 9 tiles that are each $4\frac{7}{8}$ *in. long. You want to determine about what length of wall you can tile. To quickly estimate the answer, you can round the mixed number before multiplying.*

In this lesson, you'll learn a method for estimating the value of expressions that contain fractions and mixed numbers. You'll also learn how to determine whether an estimate for an addition or multiplication expression will be less than, more than, or close to the exact answer.

Estimating with Mixed Numbers
Solve.

$$2\frac{1}{6} + 1\frac{5}{8}$$

(a) Estimate the value of the expression by rounding to the nearest $\frac{1}{2}$.

The fraction $\frac{1}{6}$ is close to 0, so round $2\frac{1}{6}$ down to 2. The fraction $\frac{5}{8}$ is close to $\frac{1}{2}$, so round $1\frac{5}{8}$ down to $1\frac{1}{2}$.

$$2 + 1\frac{5}{8} = 3\frac{1}{2}$$

(b) Is your estimate in Part (a) likely an *overestimate*, an *underestimate*, or a *close estimate*? Explain.

The estimate is likely to be an *underestimate* because each addend was rounded down to a lesser number.

(c) What is the exact value of the expression? Explain how your answers to Parts (a) and (b) can help you check your answer.

$$2\frac{1}{6} + 1\frac{5}{8} = 2\frac{4}{24} + 1\frac{15}{24} = 3\frac{19}{24}$$

The exact answer is relatively close to the estimate. It's more than the estimate, which makes sense because the estimate is an *underestimate*.

Reset

OFFLINE WORK

- Read pages 181–182.
- Complete Problems 1–3 on page 182.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Lesson 14: CORE FOCUS: Applications with Rational Numbers

Many situations in the real world involve rational numbers, such as dealing with money, making measurements, or calculating grade point averages.

In this lesson, you'll learn how to solve problems that involve rational numbers

**Watch the video to learn how to solve problems with rational numbers. Take notes in the Student Guide as you watch.

OFFLINE WORK

- Read pages 183–184.
- Complete Problems 1–3 on page 185.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Lesson 16: Extended Problems: Real-World Application

In this lesson, you'll complete Extended Problems: Real-World Application for the Fractions unit.

Many people enjoy do-it-yourself home remodeling. Remodeling work requires a builder to use good math skills to take precise measurements and make accurate calculations. The success of the finished project depends on meticulous building practices. If you decided to help with a home remodeling project

- How would you determine the amount of supplies to buy?
- How would you compute the cost of supplies?
- How might you use math to decide which stores will give you the best price on the supplies?