Math B – Unit 6 Study Guide: Combined Operations
Be sure to watch all videos within each lesson.

Many boats can be powered by the wind, by a gas engine, or both. A hybrid automobile can run on gasoline or electric power. These combinations are very powerful. Combining addition or subtraction with multiplication or division is powerful as well. You can use equations and expressions with mixed operations to solve many complex problems.

Lesson 1: The Distributive Property

If you have 3 packages of 8 chocolates, and 3 packages of 12 caramels, there are a few different ways you can calculate that you have 60 total pieces of candy. You could calculate that you have 24 chocolates and 36 caramels and then add. Or you could calculate that 1 package of chocolates and 1 package of caramels have 20 total pieces of candy, then multiply by 3. In this lesson, you'll learn how the distributive property allows you to calculate certain expressions.

> **For all numbers *a*, *b*, and *c*:**
>
> $$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$
>
> **and**
>
> $$a \cdot (b - c) = (a \cdot b) - (a \cdot c)$$

It can also be referred to as the "Rainbow Property" ☺

$$5(x + y) = 5x + 5y)$$

Remember to keep going back to the "pot of gold" as you follow the colors of the "rainbow"

More Examples:

24(a + 5) = 24a + (24*5)

3(a − b + c) = 3a − 3b + 3c

xy(4 − w − 2 + z) = 4xy − wxy − 2xy + xyz (If you combine like terms: 2xy − wxy + xyz)

OFFLINE WORK:

- Read pages 193–195.
- Complete Problems 4–18 even, 25, 27, 32–36 even, and 37 on page 196.
  Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

A messy closet might look like a jumble of different types of things—soccer cleats, T-shirts, musical instruments, textbooks, games, and more. But if someone organized that closet, all the things that are alike would be together—all the sports equipment might be together, all the clothing, and all the school supplies. Similarly, in this lesson, you will learn how to simplify mathematical expressions by combining parts of the expression that are alike.

A term is the general name for one of the components of an addition or subtraction problem. For example, in the expression $5x + 6xy − 9$, there are three terms: $5x$, $6xy$, and 9.

Like terms have the same combination of variables, but may have different coefficients. For example, $5x$ and $2x$ are like terms, but $7y$ and $3x$ are not like terms.

Constants, or terms with no variables, such as $−8$ or 0.4, are also like terms.

When an expression contains a set of two or more like terms, you can use the distributive property to combine like terms.

Simplify.

$7 + 2w + 3 + w + 8w =$ ⬚ $11w+10$

Like terms have the same combination of variables, but may have different coefficients.

If an expression contains one or more sets of like terms, simplify by **combining like terms**.

To **combine like terms**, just add the coefficients and keep the same variables.

Combine Like Terms

Combine like terms in this expression:
$$8x − 2y + 9y + 4$$

The expression contains two like terms:
$$− 2y \text{ and } 9y$$

Therefore,
$$8x − 2y + 9y + 4 = 8x + (−2 + 9)y + 4$$
$$= 8x + 7y + 4$$

OFFLINE WORK:

- Read pages 198–200.
- Complete Problems 6–12 even, 18–22 even, 28–38 even, and 41 on pages 200–201.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

Khan Academy Video: https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-variables-expressions/cc-7th-manipulating-expressions/v/combining-like-terms-and-the-distributive-property

Identifying and Combining Like Terms:

A triangle has side lengths of $4x^2 + 2x$, $3x^2 + 4x$, and $9x^2 - 1$.
What is an expression for the perimeter of the triangle?
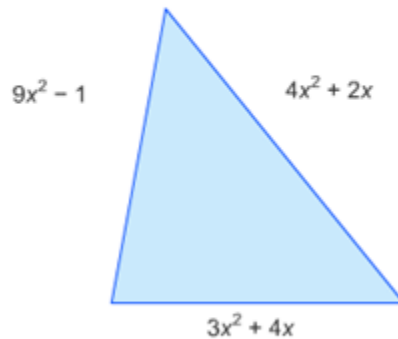
Perimeter:

$(4x^2 + 2x) + (3x^2 + 4x) + (9x^2 - 1)$

Identify Like Terms:

$(4x^2 + 3x^2 + 9x^2) + (2x + 4x) - 1$

Combine Like Terms:

$16x^2 + 6x - 1$

$9x^2 - 1$

$4x^2 + 2x$

$3x^2 + 4x$

When you subtract one variable expression from another, be sure that you subtract every term.

Ex. *Ben's age is 3x + 4 and Nolan's age is 4x – 2. How much older than Ben is Nolan?*

Identify and group like terms: $(4x - 2) - (3x + 4)$
$4x - 2 - 3x - 4$ (Notice I subtracted BOTH the 3x and 4)
$4x - 3x - 2 - 4$
$x - 6$ This is the difference between Nolan's and Ben's ages.

OFFLINE WORK:
- Read pages 202–203.
- Complete Problems 1–3 on page 204.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*

Lesson 5: Expressions with Mixed Operations

Lesson 3: Expressions with Mixed Operations

Mixed Operations means more than 1 mathematical operation in each expression:

Example: $5 \cdot 2 - (3 + 1)$ → Multiplication, Subtractions and Addition

Use PEMDAS!

PLEASE EXCUSE MY DEAR AUNT SALLY!

Parenthesis
Exponents
Multiplication and Division (from left to right)
Addition and Subtraction (from left to right)

Combine Like Terms to Solve Equations

$5m + 5 - 8m - 4 = -29$

$-3m + 1 = -29$	Combine like terms (5m – 8m and 5 – 4)
$-3m + 1 - 1 = -29 - 1$	Subtract 1 from both sides
$-3m = -30$	Simplify
$\dfrac{-3m}{-3} = \dfrac{-30}{-3}$	Divide both sides by -3
$m = 10$	

OFFLINE WORK:

- Read pages 205–208.
- Complete Problems 5–11 odd, 15, 23–29 odd, 35, and 39 on pages 208–209.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets.*

Khan Academy Help:
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-variables-expressions/cc-7th-2-step-equations/v/why-we-do-the-same-thing-to-both-sides-two-step-equations

Five friends are planning an event and will split the cost evenly. Entertainment costs $55.75. The caterer costs $25and and additional $4.25 per guest. Gift bags cost $3.50 per guest.

Write an algebraic expression for the event cost. *(Let x be the number of guests)*

55.75 + (25 + 4.25x) + 3.50x

Write an algebraic expression for each friend's share of the event cost.

The five friends split the cost evenly.

$$\frac{55.75 + (25 + 4.25x) + 3.50x}{5}$$

How much will each friend pay if they invite 30 guests?

$$\frac{55.75 + (25 + 4.25x) + 3.50x}{5}$$

$$= \frac{55.75 + (25 + 4.25 \cdot 30) + 3.50 \cdot 30}{5}$$ Substitute 30 for x and simplify.

$$= \frac{55.75 + (25 + 127.50) + 105}{5}$$ Simplify inside the parentheses.

$$= \frac{55.75 + 152.50 + 105}{5}$$ Simplify the numerator.

$$= \frac{313.25}{5}$$ Divide the numerator by the denominator.

$$= 62.65$$

Each friend will pay $62.65.

OFFLINE WORK:

- Read pages 210–212.
- Complete Problems 1–3 on page 212.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets.*

Lesson 8: Equations with Mixed Operations

Before starting to solve the equation, simplify one or both sides of the equation by combining like terms or removing parentheses.

- **Just remember that you must undo the operations using the reverse order of operations.**

You must get the variable alone (isolate the variable) on one side of the equal sign; **get rid of the numbers on the same side as the variable by doing the opposite operation**

Example: Solve for c:

$3c + 6 = 30$

$3c + 6 - 6 = 30 - 6$ 6 is being added to c, so SUBTRACT 6 from both sides:

$3c = 24$

$\dfrac{3c}{3} = \dfrac{24}{3}$ 3 is being multiplied by c, so DIVIDE 3 into both sides

$c = 8$

OFFLINE LEARNING:

- Read pages 213–217.
- Complete Problems 3–15 odd, 19–29 odd, 33, and 35 on pages 217–218.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*.

To solve a multistep equation

Step 1 Simplify both sides of the equation.

Step 2 Isolate the variable on one side of the equation.

Step 3 Check your answer.

Solve the equation.

$$5k + 4 + 3k = -12$$
$$8k + 4 = -12$$ Combine like terms.
$$8k + 4 - 4 = -12 - 4$$ Subtract 4 from both sides.
$$8k = -16$$
$$8k \div 8 = -16 \div 8$$ Divide both sides by 8.
$$k = -2$$

Check the solution.

$$5k + 4 + 3k = -12$$
$$5 \cdot (-2) + 4 + 3 \cdot (-2) = -12$$
$$-10 + 4 + (-6) = -12$$
$$-6 - 6 = -12$$
$$-12 = -12$$

OFFLINE LEARNING:

- Read pages 219–220.
- Complete Problems 1–3 on page 221.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets.*

Khan Video https://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities/complicated_equations/v/solving-equations-with-the-distributive-property

https://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities/solving_for_variable/e/solving_for_a_variable

Lesson 10: Inequalities

Inequality: compares numbers or expressions using the symbols:

> Greater Than

< Less Than

≥ Greater Than or Equal to

≤ Less Than or Equal to

Replacement Set: a list of possible numbers that will make the inequality true

Solution Set: the set of all numbers that will make the inequality true

Example: **Inequality**: $x \geq 4$ some number (x) is greater than or equal to 4

Replacement Set: {-4, -3, 2, 4, 5, 7}

Solution Set: {4, 5, 7}   numbers greater than or equal to 4

Graphing Inequalities
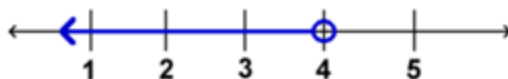
1. Solve the inequality
2. Graph the solution

Example: $5x - 10 < 10$

$5x - 10 + 10 < 10 + 10$

$5x < 20$

$x < 4$

✓ When you're solving an inequality, simplify the inequality using transformations as you would when solving an equation.

✓ Remember, when you're multiplying or dividing both sides of the inequality by a negative number, reverse the direction of the inequality.

✓ When you graph the solution to the inequality on a number line, use open circles to indicate < or > relationships, and filled-in circles to indicate ≤ or ≥ relationships.

OFFLINE LEARNING:

- Read pages 222–226 in the reference guide.
- Complete Problems 3–5, 11–19 odd, 27, and 29 on pages 226–227.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets*

Lesson 11: Core Focus: Applications of Inequalities

Clue Words
- "less than" (<)
- "greater than" (>)
- "less than or equal to" (≤)
  - at most
  - maximum
  - no more than
- "greater than or equal to" (≥)
  - at least
  - minimum
  - no less than

Example: Take note of the use of the phrase "less than" in the following example.

Janelle must complete an obstacle course in less than 300 s to win first place. It takes a total of 60 s to move between all 8 obstacles. How quickly must Janelle complete each obstacle to win?

$$8x + 60 < 300$$
$$8x + 60 - 60 < 300 - 60$$
$$8x < 240$$
$$8x \div 8 < 240 \div 8$$
$$x < 30$$

Janelle must complete each obstacle in less than 30 s to win.

OFFLINE LEARNING:

- Read pages 228–229 in the reference guide.
- Complete Problems 1–3 on page 229.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics B: A Reference Guide and Problem Sets.*

Khan Academy: https://www.khanacademy.org/math/algebra-basics/core-algebra-linear-equations-inequalities/core-algebra-linear-inequalities/v/multi-step-inequalities