Lesson 1 – Semester 1 Introduction

Lesson 2 – Expressions

BE SURE TO REVIEW THE OLS LESSON!! There are a lot of important details to remember.

Groundwork

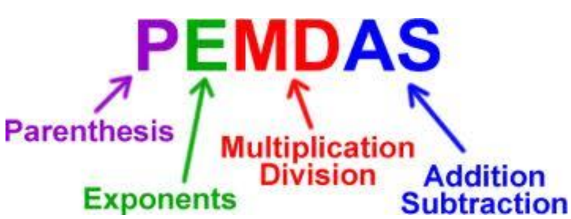
Sometimes expressions include powers. You should remember that a power, such as 3^2, has two parts: a base and an exponent. In this case, the base is 3 and the exponent is 2. The exponent tells you how many times you should multiply the base by itself. So,

$$3^2 = 3 \bullet 3 = 9$$

When the base is 10 the exponent is called a *power of 10*.

standard form	1,000
as a product of tens	10 × 10 × 10
exponential form	10^3
word form	one thousand

REMEMBER:



Using the Order of Operations
The order of operations is the sequence you must follow to simplify expressions with more than one operation.

1. Parentheses
2. Exponents
3. Multiplication and Division (left to right)
4. Addition and Subtraction

$$\frac{20 + 2(3 + 5)}{16 - 49 \div 7}$$

Fraction Bar

1. Simplify the Numerator
2. Simplify the Denominator
3. Simplify the Fraction

Simplify.

$3^2 + 4(7 + 2)$

Simplify within parentheses.

$3^2 + 4(7 + 2) = 3^2 + 4(9)$	Add 7 and 2 within the parentheses.

Simplify exponents.

$= 9 + 4(9)$	Simplify 3^2.

Multiply and divide.

$= 9 + 36$	Multiply 4 by 9.

Add and subtract.

$= 45$	Add 9 and 36.

When simplifying expressions, you must follow the order of operations.

- Evaluate grouping symbols from the "inside out."
- Work through the top and bottom of a fraction (the numerator and denominator) as if they were in parentheses.
- Calculate powers.
- Multiply and divide from left to right.
- Add and subtract from left to right.

OFFLINE WORK:

- Read pages 7–9 in the reference guide.
- Complete Problems 1–17 odd and 26–28 on pages 9–10.
- Complete Problems 2–24 even on pages 9–10 for extra practice (optional).
- For a challenge, complete Problems 31 and 32 on page 10 (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 3: Distributive Property

For all real numbers, a, b, and c,

$$a(b + c) = ab + ac$$

EXAMPLE: 8(x – 4) = 8x – (8*4)

Like terms are terms that have the same variables raised to the same powers.

All constants are like terms.

Like Terms
Identify the like terms in the expression.

$$x^3y + 4x^3 + x^3 + 7x^3y$$

x^3y and $7x^3y$ are like terms.
$4x^3$ and x^3 are like terms.

Always distribute, if possible, before looking for like terms.

Simplify.

$20(x + 3) + 13x$

Distribute.

$20x + 60 + 13x$ — Multiply 20 by x and 20 by 3.

Reorder.

$20x + 13x + 60$ — Use the commutative property to reorder the terms.

Combine like terms.

$33x + 60$ — Combine $20x$ and $13x$ by adding the coefficients and keeping the variable part.

REMEMBER: Watch all videos and complete the skills in the OLS

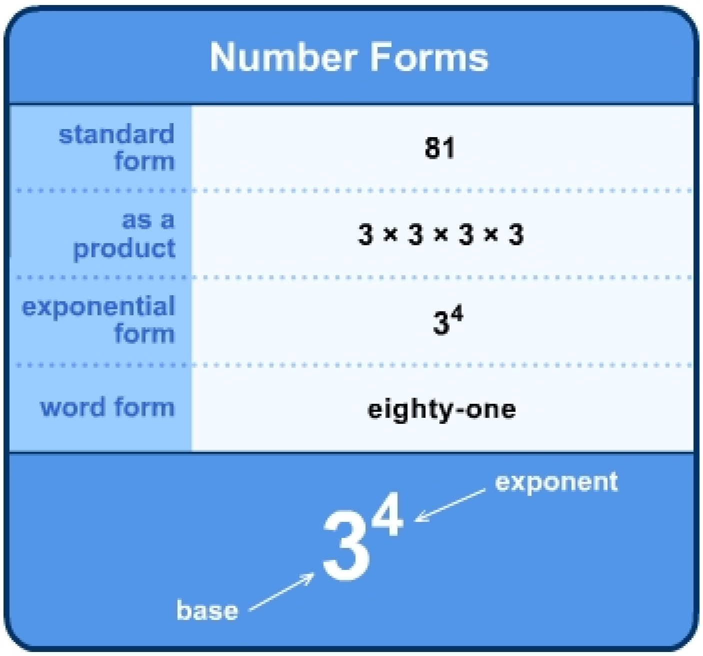
OFFLINE WORK:

- Read pages 11–13 in the reference guide.
- Complete Problems 1–15 odd and 17–23 all on page 14
- Complete Problem 28 on page 14 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 4 – Optional Your Choice to SKIP

Lesson 5: Positive Exponents

base: a number that is raised to some power (for example, in 5^3, 5 is the base)
exponent: a number that shows how many times the base is used as a factor
power of a number: the product when a number is multiplied by itself a given number of times (e.g., 5^3 or 5 • 5 • 5 is the third power of five)
factor: any of two or more numbers multiplied to form a product



REMINDER: Be sure to follow **Order of Operations**

Parentheses
Exponents
Multiply or **D**ivide
Add or **S**ubtract

$$24+18\div 9 * 3-14$$

Multiply and divide.

$24 + 18 \div 9 \cdot 3 - 14 = 24 + 2 \cdot 3 - 14$	Divide 18 by 9 first.
$= 24 + 6 - 14$	Multiply 2 by 3.

Add and subtract.

$= 30 - 14$	Add 24 and 6.
$= 16$	Subtract 14 from 30 to find the answer.

What if the exponent is a number like 0 or 1? Use the properties of exponents 0 and 1. According to these properties,

- **Any number raised to 0 is 1.**
- Any number raised to 1 is simply the number.

  For all nonzero real numbers a:

  $$a^0 = 1$$

  $$a^1 = a$$

-

SPECIAL REMINDERS:

$-\mathbf{3}^2 = -1 * \mathbf{3} * \mathbf{3} = -9$ *(The negative sign is not squared)*

$(\mathbf{-3})^2 = \mathbf{-3} * \mathbf{-3} = 9$ *(Since the negative sign is in the parentheses, it is also squared)*

OFFLINE WORK:

- Read pages 15–18 in the reference guide.
- Complete Problems 1–29 odd and 32 on pages 18–19.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 6: Negative Exponents

Properties of Negative Exponents

For all nonzero numbers *a* and integers *n*

$$a^{-1} = \frac{1}{a}$$

$$a^{-n} = \frac{1}{a^n} = \frac{1}{\underbrace{a \cdot a \cdot a \cdot a}_{n \text{ factors}}}$$

$$\frac{1}{a^{-n}} = a^n = \underbrace{a \cdot a \cdot a \cdot a}_{n \text{ factors}}$$

See if you can simplify these powers. **Remember to use the properties of negative exponents to rewrite each power with a positive exponent.**

Properties of Negative Exponents

For all nonzero *a* and integer *n*,

$a^{-1} = \frac{1}{a}$ $\qquad a^{-n} = \frac{1}{a^n}$ $\qquad \frac{1}{a^{-n}} = a^n$

Examples:

$4^{-1} = \frac{1}{4}$ $\qquad 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$ $\qquad \frac{1}{5^{-3}} = 5^3 = 125$

Look at these more complicated expressions. How can you simplify them?

You can simplify these expressions the same way you simplify expressions with positive exponents:

- Use properties of exponents
- Follow the order of operations

- $\frac{10}{5^{-3}}$

$\frac{10}{5^{-3}} = 10 \cdot \frac{1}{5^{-3}}$	Write the fraction as the product of a whole number and a fraction.
$= 10 \cdot 5^3$	Write the expression using a positive exponent.
$= 10 \cdot 125$	Evaluate 5^3.
$= 1{,}250$	Multiply.

$\frac{6^{-2}}{2^{-5}}$

$\frac{6^{-2}}{2^{-5}} = 6^{-2} \cdot \frac{1}{2^{-5}}$	Write the fraction as a product.
$= \frac{1}{6^2} \cdot 2^5$	Write the expression using positive exponents.
$= \frac{1}{36} \cdot 32$	Evaluate the powers.
$= \frac{32}{36}$	Multiply.
$= \frac{8}{9}$	Express in lowest terms.

You can use the properties of exponents to solve equations with exponents.
View these examples.

Solve for x.

$$5^x = 25$$
$$5^x = 5^2$$
$$x = 2$$

Solve for x.

$$5^x = \frac{1}{25}$$
$$5^x = \frac{1}{5^2}$$
$$5^x = 5^{-2}$$
$$x = -2$$

BE SURE TO WATCH ALL VIDEOS IN THE ONLINE SCHOOL

OFFLINE WORK:

- Read pages 20–22 in the reference guide.
- Complete Problems 1–8 and 22–25 on pages 22–23.
- Complete Problems 9–13 and 26–27 on pages 22–23 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 7: Optional Your Choice to SKIP

Lesson 8: CORE FOCUS: Working with Exponents

PROPERTY OF EXPONENTS

- **The Product of Powers Property** states that when you multiply powers with equal bases, you must **add** the exponents.
- $2^5 * 2^6 = 2^{11}$

- **The Quotient of Powers Property** states that when you divide powers with equal bases, you must **subtract** the exponent in divisor from the exponent in the dividend.

$$\frac{7^6}{7^{-2}} = 7^8$$

- $\dfrac{3^9}{3^4} = 3^5$
- **The Power to a Power Property** states that when you raise a power to a power, you must **multiply** the exponents.

$$(9^5)^3 = 9^{15}$$

- $(4^{-3})^4 = 4^{-12}$

Simplifying Expressions Involving Powers

Simplify $2^4 \cdot (2^3)^2 \cdot 4$.

$2^4 \cdot (2^3)^2 \cdot 4$	$= 2^4 \cdot 2^6 \cdot 4$	Power of a Power Property
	$= 2^{10} \cdot 4$	Product of Powers Property
	$= 1024 \cdot 4$	Simplify.
	$= 4096$	Simplify.

OFFLINE WORK:

- Read pages 24–25 in the reference guide.
- Complete Problems 1–12 on page 26.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 9: Scientific Notation

At any moment, there are an estimated 10,000,000,000,000,000,000 live insects in the world. Because this number has so many zeros, it would be easy to make a mistake when copying it down. Scientists often work with numbers that are either very large or very small and have many zeros. They do not lose time copying zeros because they write the numbers in scientific notation

Scientific notation is based on multiplying numbers by powers of 10.

Find the product of a decimal and power of 10.

$$3.5 \times 10^5$$

Step 1: Evaluate 10^5

$$10^5 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000$$

Step 2: Multiply 3.5×10^5

$$3.5 \times 10^5 = 3.5 \times 100{,}000$$
$$= 350{,}000$$

Power Properties

Product of Powers Property

$$a^m \cdot a^n = a^{m+n}$$

Example:

$$10^5 \cdot 10^{-1} = 10^{5+(-1)} = 10^4$$

Quotient of Powers Property

$$\frac{a^m}{a^n} = a^{m-n}$$

Example:

$$\frac{10^7}{10^{-3}} = 10^{7-(-3)} = 10^{10}$$

A number written in **scientific notation** is a product of two numbers.

- **The first number** is greater than or equal to 1 but is less than 10.
- **The second number** is a power of 10.

$$0.000009 = 9 \times 10^{\boxed{-6}}$$

$$30{,}400{,}000{,}000 = \boxed{3.04} \times 10^{\boxed{10}}$$

Adding when exponents are the same:

Add. Write the result in scientific notation.

$3.6 \times 10^8 + 2.45 \times 10^8$

The exponents of 10 are the same. Use the distributive property to add.

$$\begin{aligned} 3.6 \times 10^8 + 2.45 \times 10^8 &= (3.6 + 2.45) \times 10^8 \\ &= 6.05 \times 10^8 \end{aligned}$$

Adding when exponents are NOT the same:

Add. Write the result in scientific notation.
$1.5 \times 10^7 + 2.6 \times 10^8$

Choose a number to rewrite.

$1.5 \times 10^7 + 2.6 \times 10^8$

To add, the exponents must be the same.

Choose to rewrite the first number with an exponent of 8.

Rewrite the number.

1.5×10^7

$0.15 \times 10^1 \times 10^7$

0.15×10^8

To increase the exponent from 7 to 8, you need to add a factor of 10.

Rewrite 1.5 as a number times 10.

Apply the distributive property.

$0.15 \times 10^8 + 2.6 \times 10^8$

$(0.15 + 2.6) \times 10^8$

2.75×10^8

Rewrite the original problem and apply the distributive property.

Multiplying: (multiply the first numbers then multiply the 10's)

Multiply. Write the result in scientific notation.

$(4.03 \times 10^5)(2.5 \times 10^9)$

Use the commutative and associate properties to regroup and reorder the numbers.

$(4.03 \times 2.5)(10^5 \times 10^9) = 10.075 \times 10^{5+9}$
$= 10.075 \times 10^{14}$

Write the result in scientific notation.

$= 1.0075 \times 10^1 \times 10^{14}$
$= 1.0075 \times 10^{15}$

Dividing: (Divide the first numbers then divide the 10's)

Divide. Write the result in scientific notation.

$$\frac{9 \times 10^{15}}{3 \times 10^{8}}$$

$$\frac{9}{3} \times \frac{10^{15}}{10^{8}} = 3 \times 10^{15-8}$$

$$= 3 \times 10^{7}$$

OFFLINE WORK:

- Read pages 27–29.
- Complete Problems 1–33 odd on pages 29–30.
- Complete Problems 2–32 even on pages 29–30 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 10: Optional Your Choice to Skip

Lesson 11: Orders of Magnitude

There are different ways to estimate numbers. One way to estimate is to round. In this lesson, you will combine what you know about rounding and what you know about writing numbers in scientific notation to estimate numbers as the product of a single digit and a power of 10.

In addition, you will learn how to estimate a number as just a power of 10. This type of rough estimate is called an order-of-magnitude estimate.

Estimate the number as the product of a single digit and a power of 10.

18,300,000,000,000,000

Write the number in scientific notation.	1.83×10^{16}
Round the decimal to the nearest whole number.	2×10^{16}
Check that the whole number is a single digit. Otherwise, move the decimal point and adjust the power of 10.	2×10^{16}

Summary

Determine the best order-of-magnitude estimate for 4300.

Step 1: Determine the two nearest powers of 10 that 4300 lies between.

$10^3 < 4300 < 10^4$

Step 2: Determine the power of 10 that 4300 is nearer to.

3300 < 5700, so 4300 is nearer to 1000, or 10^3.

Step 3: State the best order-of-magnitude estimate.

The best order-of-magnitude estimate is 10^3, or 1000.

OFFLINE WORK:

- Read pages 31–33 in the reference guide.
- Complete Problems 1–19 odd on page 33.
- Complete Problems 2–18 even on pages 33 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

<u>Lesson 12: CORE FOCUS: Working with Scientific Notation</u>

Light travels at a speed of about 3×10^8 m/s. How can you find the number of kilometers light travels in a year? How could you find the number of years it would take for light to travel 1×10^{15} km?

In this lesson, you will learn how to solve real-world problems that use numbers expressed in scientific notation and you will use a process called dimensional analysis to convert units.

BE SURE TO WATCH THE VIDEOS IN OLS

Dimensional Analysis

Convert 8.2×10^8 m/s to kilometers per hour.

Step 1: Convert seconds to hours.

$$\frac{8.2\times10^8\ \text{m}}{1\ \text{s}} = \frac{8.2\times10^8\ \text{m}}{1\ \text{s}} \bullet \frac{3600\ \text{s}}{1\ \text{h}}$$

$$= \frac{8.2\times10^8\ \text{m}}{1\ \text{s}} \bullet \frac{3.6\times10^3\ \text{s}}{1\ \text{h}} \qquad = \frac{29.52\times10^{11}\ \text{m}}{1\ \text{h}}$$

Convert 8.2×10^8 m/s to kilometers per hour.

Step 2: Convert meters to kilometers.

$$\frac{2.952\times10^{12}\ \text{m}}{1\ \text{h}} = \frac{2.952\times10^{12}\ \text{m}}{1\ \text{h}} \bullet \frac{1\ \text{km}}{1000\ \text{m}}$$

$$= \frac{2.952\times10^{12}\ \text{m}}{1\ \text{h}} \bullet \frac{1\ \text{km}}{1\times10^3\ \text{m}}$$

$$= \frac{2.952\times10^{12}\ \text{km}}{1\times10^3\ \text{h}} = \frac{2.952\times10^{9}\ \text{km}}{1\ \text{h}}$$

Convert 8.2×10^8 m/s to kilometers per hour.

$$8.2\times10^8\ \text{m/s} = 2.952\times10^9\ \text{kmph}$$

OFFLINE WORK:

- Read pages 34–36 in the reference guide.
- Complete Problems 1–4 on page 37.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 13: Comparing Big and Small Numbers

How many times greater is the population of India than the population of Sweden? How many times greater is the distance between the sun and the earth than the distance between the earth and the moon? How many times greater is the mass of a caffeine molecule than the mass of a water molecule?

All of these questions involve finding the number of times one very large or very small number is greater than another very large or very small number. This lesson will give you the tools to solve these types of problems.

The Core Concept

Expressing very large or small numbers in scientific notation, as the product of a whole number and a power of 10, or as an order of magnitude, allows us to more easily make comparisons or perform operations.

Overview

Estimate the number of times 62,300,000,000 is greater than 460,000,000.

$$62{,}300{,}000{,}000 = 6.23 \times 10^{10} \approx 6 \times 10^{10}$$

$$460{,}000{,}000 = 4.6 \times 10^{8} \approx 5 \times 10^{8}$$

$$6 \times 10^{10} > 5 \times 10^{8}$$

Divide the greater by the lesser value:

$$\begin{aligned}\frac{6 \times 10^{10}}{5 \times 10^{8}} &= \frac{6}{5} \times \frac{10^{10}}{10^{8}} \\ &= 1.2 \times 10^{10-8} \\ &= 1.2 \times 10^{2} \\ &= 1.2 \times 100 \\ &= 120\end{aligned}$$

62,300,000,000 is about 120 times greater than 460,000,000.

OFFLINE WORK:

- Read pages 38–39 in the reference guide
- Complete Problems 1–4 on page 39.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.