## Lesson1: Equations in Two Variables

Solution to a $\mathbf{2}$ Variable Equation: an ordered pair ( $x, y$ ); in other words, 2 numbers that make the equation true

Example: $\quad 3 x+8 y=38 \quad$ Is $(2,4)$ a solution?
Substitute 2 for $x$ and 4 for $y$

$$
\begin{array}{ll}
3(2)+8(4)=38 & \\
6+32=38 & \\
38=38 & \text { This is true, so }(2,4) \text { is a solution }
\end{array}
$$

## To Find a Solution to a $\mathbf{2}$ Variable Equation:

Choose any value for $x$, solve for $y$
Example: $6 x+3 y=9$

1. Choose $x=2$ :

\[

\]

Solution: (2,-1)
To Solve a 2 Variable Equation for y :
Use transformations to get y by itself (usually on the left side)
Example:
$6 x+2 y=-8$
$6 x-6 x+2 y=-8-6 x$ Subtract $6 x$ from both sides
$2 y=-8-6 x$
$\underline{2 y}=\underline{-8}-\underline{6 x} \quad$ Divide EVERY term by 2
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$y=-4-3 x$

## OFFLINE WORK

- Read pages 83-85.
- Complete Problems 1-15 odd, 27, 29 on pages 86-87.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of Intermediate Mathematics C: A Reference Guide and Problem Sets.


## Lesson 2: Graphs

## Coordinate Plane

Horizontal line is the $x$-axis; Vertical line is the $y$-axis
The point where the 2 lines meet is the origin $(0,0)$
The axes divide the plane into 4 quadrants, numbered I, II, III, IV


## Ordered Pair

Has the form ( $\mathrm{x}, \mathrm{y}$ ) - x and y represent numbers that are called coordinates
To locate the ordered pair on the coordinate plane:
Start at the origin $(0,0)$

## YOU HAVE TO CRAWL BEFORE YOU CAN STAND!!

First, move left or right on the x-axis: CRAWL FIRST
Second, move up or down on the $y$-axis: STAND SECOND


Example:


Point A: $(6,8)$ Right 6, Up 8
Point B: (-6,2) Left 6, Up 2
Point C: (4,-4) Right 4, Down 4
Positive x-coordinate - move RIGHT
Negative x-coordinate - move LEFT
Positive y-coordinate - move UP
Negative y-coordinate - move DOWN

- Read pages $88-89$ in the reference guide.
- Complete Problems 1-25 and 27-28 on pages 90-92.
- Complete Problems 26, 29, 30 on pages 91-92 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of Intermediate Mathematics C: A Reference Guide and Problem Sets.


## Lessons 3: Lines and Intercepts

Co-linear: points that, when connected, form a straight line
To determine if points are Co-linear: plot the points
draw a straight line through them
Standard Form of a linear equation: $\mathbf{A x}+\mathbf{B y}=\mathbf{C}$
$A, B$ and $C$ are integers; $A$ and $B$ do not equal 0
Example: $\quad 6 x+2 y=16: \quad A=6, B=2, C=16$

Graph a Line on a Coordinate Plane

1. Start with 2 ordered pairs: A: $(1,1) \mathrm{B}:(2,3)$
2. Plot each point on a coordinate plane
3. Draw a straight line through the points


## Intercepts of a Line

$x$-intercept - the $x$-coordinate of the point where the line crosses (intersects) the $x$-axis. Find the $x$-intercept by substituting $\mathbf{0}$ for $\mathbf{y}$ in the equation
$y$-intercept - the $y$-coordinate of the point where the line crosses (intersects) the $y$-axis. Find the $y$-intercept by substituting $\mathbf{0}$ for $\mathbf{x}$ in the equation
$x$-intercept $=(2,0)$
$y$-intercept $=(0,5)$


## Graph the equation of a line using Intercepts

Example: $\quad 5 x-4 y=20$
$5 x-4(0)=20 \quad$ Find the $x$-intercept by substituting 0 for $y$
$5 x=20 \quad$ Divide both sides by 5
$x=4$
$x$-intercept is $(4,0)$
$5(0)-4 y=20 \quad$ Find the $y$-intercept by substituting 0 for $x$
$-4 y=20 \quad$ Divide both sides by -4
$y=-5$
$y$-intercept is ( $0,-5$ )
Plot $(4,0)$ and $(0,-5)$ on a coordinate plane and draw a straight line through the points

## OFFLINE WORK

- Read pages $93-96$ in the reference guide.
- Complete Problems 1-10 and 11-27 odd on pages 96-97.
- Complete Problems 12-24 even on pages 96-97 (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of Intermediate Mathematics C: A Reference Guide and Problem Sets


## Lesson 4: SKIP (Your Choice)

## Lesson 5: Slope

Slope: A measurement of the steepness of the line
Slope $=\frac{\text { rise }}{\text { run }}=\frac{\text { verticalchange }}{\text { horizonalchange }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

$$
\text { Slope }=\frac{\text { rise }}{\text { run }}=\frac{-3}{7}
$$




Positive Slope - line goes up from left to right
Negative Slope - line goes down from left to right
Zero Slope - horizontal line
Undefined Slope - vertical line
Example: Find the slope of the line that passes through the points $(-3,2)$ and $(1,1)$


1. Label the points: $(-3,2)$ and $(1,1)$

$$
\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right) \quad\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)
$$

2. Substitute the numbers into the slope formula: $\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{1-2}{1-(-3)}=$ $\frac{-1}{4}$
3. This line has a slope of $\frac{-1}{4}$


MR. SLOPE GLY

## Lesson 6: Simple Linear Graphs

The origin of the coordinate plane is the point where the $x$ - and $y$-axes intersect. You use the origin, or point $(0,0)$, to describe the location of other points on the graph. For instance, the point $(3,-4)$ is 3 units to the right of the origin and 4 units down.

Suppose you drew a line that passes through $(0,0)$ and $(3,-4)$. That line would pass through the origin. In this lesson, you will learn how to write an equation of a line that passes through the origin.

For lines that pass through the origin, both the $x$-intercept and the $y$-intercept are zero.

## Determining the Equation of a Graphed Line

Identify another point on the line and substitute into the point-slope formula. Then solve for $m$.

$$
\begin{aligned}
& m=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}=\frac{y-0}{x-0}=\frac{y}{x} \\
& m=\frac{y}{x}
\end{aligned}
$$



Solve for $y$.

$$
\begin{aligned}
m & =\frac{y}{x} \\
m \cdot x & =\frac{y}{x} \cdot x \\
m x & =y \\
y & =m x
\end{aligned}
$$

Equation of Line That Passes Through the Origin

$$
y=m x
$$

m = slope of the line

Find the $x$ and $y$ intercepts, then use the slope formula to find the slope

```
Example: \(\quad 5 x-4 y=20\)
\(5 x-4(0)=20 \quad\) Find the \(x\)-intercept by substituting 0 for \(y\)
\(5 x=20 \quad\) Divide both sides by 5
\(x=4 \quad x\)-intercept is \((4,0)\)
\(5(0)-4 y=20 \quad\) Find the \(y\)-intercept by substituting 0 for \(x\)
\(-4 y=20 \quad\) Divide both sides by -4
    \(y=-5 \quad y\)-intercept is \((0,-5)\)
```

Find the slope of the line that passes through $(4,0)$ and $(0,-5)$
$\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{-5-0}{0-4}=\frac{-5}{-4}=\frac{5}{4}$

## OFFLINE WORK:

- Read pages 104-106.
- Complete Problems 7-9, 13-16, and 23-25 on pages 107-108.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of Intermediate Mathematics C: A Reference Guide and Problem Sets.


## LESSON 7: SKIP (Your Choice)

## Lesson 8: Using Slope as a Rate

Slope can be interpreted as a rate: a comparison between 2 units
Examples of Rates: feet climbed per hour gallons of gas used per minute

Unit Rate: the second term represents 1 Unit
Example: $120 \mathrm{mph}=120$ miles in $\mathbf{1}$ hour - this is a unit rate

Slope $=\frac{\text { rise }}{\text { run }} \quad$ Rise $=$ units on the y -axis

$$
\text { Run }=\text { units on the } x \text {-axis }
$$

Example:
Ross is hiking at a steady rate of 2.5 miles per hour. Construct a graph corresponding to this rate.

| Hours | Miles |
| :---: | :---: |
| 1 | 2.5 |
| 2 | 5 |
| 3 | 7.5 |
| 4 | 10 |
| 5 | 12.5 |



Choose any point on the $y$-axis for the rise and any point on the $x$-axis for the run
The slope of this line $=\frac{\text { rise }}{\text { run }}=\frac{2.5}{1}=\frac{5}{2}=\frac{12.5}{5}=2.5$
Interpret the slope as a Rate:
Ross is hiking at a rate of 2.5 miles per hour

## OFFLINE WORK

- Read pages 109-112.
- Complete Problems 1-11 odd, 12-13, and 15-19 odd on pages 113-114.
- Complete Problems 2-10 even 14, and 16-20 even on pages 113-114 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of Intermediate Mathematics C: A Reference Guide and Problem Sets


## Lesson 9: Slope and Similar Triangles

Similar Triangles: the angles are congruent and the sides are proportional


$$
\begin{aligned}
& \frac{A B}{D E}=\frac{6}{3}=2 \\
& \frac{B C}{E F}=\frac{4}{2}=2
\end{aligned}
$$



The angles are congruent and the sides all have the same proportion of 2, therefore, these are similar triangles and their hypotenuses have the same slope

Example:


Since the 2 triangles are similar, the slope of the hypotenuse is the same everywhere on the line. You can choose any $\mathbf{2}$ points on the line to determine the slope of the line

## OFFLINE WORK

- Read pages 115-116 in the reference guide.
- Complete Problems 1-8 on page 117.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of Intermediate Mathematics C: A Reference Guide and Problem Sets.


## Lessons 10: Comparing Proportional Relationships

Proportional Relationship: a relationship that can be described by an equation in the form of: $y=k x$ where $k$ is a constant number, the slope or the rate

Example: $\quad y=14 x$
14 is the coefficient of $x$
14 is the rate
14 is the slope
Example of Comparing Slopes and Rates:

Clyde's Junkyard and Viola's Junkyard both crush cars and sell them for scrap. To track how many cars her company crushes, Viola uses the equation $c=21 h$, where $h$ represents hours and $c$ represents the number of cars crushed. Clyde uses a graph to keep track of how many cars his company crushes.

Which company crushes cars at a faster rate?
Step 1: Find how many cars Viola's company crushes per hour.

Since $c=21 h$, the slope is 21 . Therefore, her company crushes 21 cars per hour.
Step 2: Find how many cars Clyde's company crushes per hour.

$$
m=\frac{100-20}{5-1}=\frac{80}{4}=20
$$

The slope is 20 , so his company crushes 20 cars per hour.

Step 3: Compare the rates.
Since $21>20$, Viola's company crushes cars at a faster rate.


## OFFLINE WORK

- Complete Problems 1-10 on pages 121-123.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of Intermediate Mathematics C: A Reference Guide and Problem Sets.


## Lessons 11: Equations from Graphs

## Write the equation of a line from a graph:

1. Identify 2 points on the line
2. Use the slope formula to find the slope: slope $=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
3. Write the equation in the requested form

Example:


1. Point $A=(1,1)$ Point $B=(3,2)$
2. Slope $=\frac{2-1}{3-1}=\frac{1}{2}$
3. Point Slope Form:

Standard Form:
Slope-Intercept Form:

$$
\begin{aligned}
& y-1= 1 / 2(x-1) \\
&-x+2 y=1 \\
& y=1 / 2 x+1 / 2
\end{aligned}
$$

Equation of a Horizontal Line: the y coordinate of every point on a horizontal line is the same. The slope of a horizontal line is 0 .
Example: $\mathrm{y}=8$


Equation of Vertical Line: the $x$ coordinate of every point on a vertical line is the same. The slope of a vertical line is undefined.

Example: $x=3$


- Read pages 124-125.
- Complete Problems 1-2 on page 125.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of Intermediate Mathematics C: A Reference Guide and Problem Sets.


## Lesson 12: REVIEW

## Lesson 13: OLS Unit Test

## Lessons 14: Applications: Linear Models

Linear Model: 1 . The rate is constant, meaning the slope is the same everywhere on the line
2. The equation can be written in Slope-Intercept Form
3. The graph is a straight line

Example: Damian starts with $\$ 100$ in his savings account. Every month, he deposits another $\$ 25$. Is this a linear model?
The balance in his account is given by the equation:

$$
B=25 t+100 \text { ( } t=\text { number of months) }
$$

1. Make a table showing his balance at 1, 4, 9 and 12 months

| $t$ | 1 | 4 | 9 | 12 |
| :---: | :---: | :---: | :---: | :---: |
| B | $\begin{aligned} B & =25 t+100 \\ & =25 \cdot 1+100 \\ & =25+100 \\ & =125 \end{aligned}$ | $\begin{aligned} B & =25 t+100 \\ & =25 \cdot 4+100 \\ & =100+100 \\ & =200 \end{aligned}$ | $\begin{aligned} B & =25 t+100 \\ & =25 \cdot 9+100 \\ & =225+100 \\ & =325 \end{aligned}$ | $\begin{aligned} B & =25 t+100 \\ & =25 \cdot 12+100 \\ & =300+100 \\ & =400 \end{aligned}$ |

2. Plot the ordered pairs:
$(1,125)$
$(4,200)$
$(9,325)$

3. Yes, this is a linear model

The slope is the same everywhere on the line
The graph is a straight line - the rate of change is constant

