IMC – Unit 4 *LINES* Study Guide

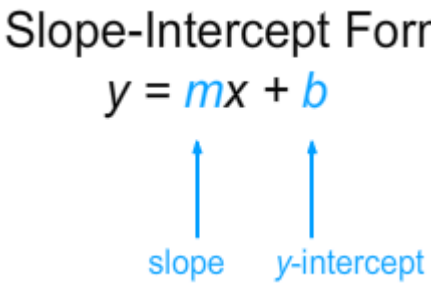
In this unit, you will continue to explore linear equations in two variables. You will learn how to find the equation of a line from its graph. You will then learn how to write equations that model real-world problems.

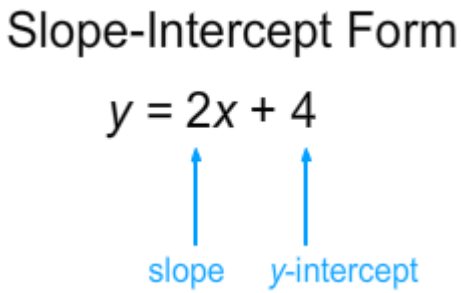
Lesson 1 – Slope Intercept Form

The first time you walked from your house to a friend's home, you probably just wanted to find your way there. After you visited your friend a few times, you might have tried to find a route that was quicker or more direct. Such a shortcut could save you time and effort. The same is true in algebra. Although you already now know how to find the slope of a linear equation, learning a quicker, shorter method could save you a great deal of time and effort.

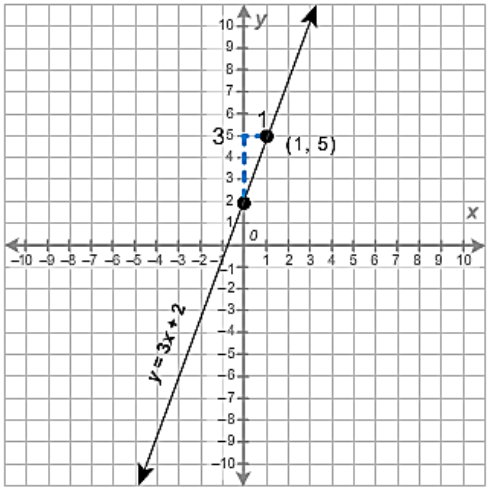
slope-intercept form of an equation of a line

the equation of a line in the form $y = mx + b$, where m is the slope and b is the y-intercept



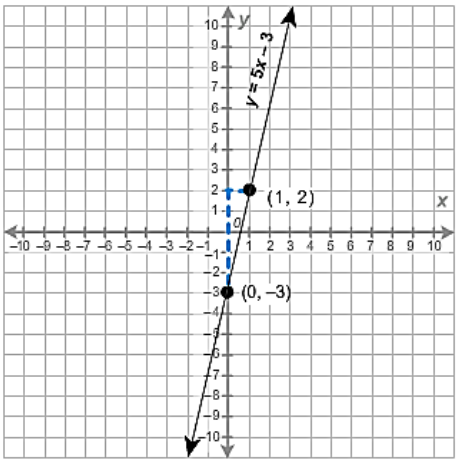


BE SURE TO WATCH ALL OLS VIDEOS



$y = 3x + 2$

$\text{slope} = 3 = \frac{3}{1}$



$$y = 5x - 3$$
$$y = 5x + {}^{-}3$$
$$\text{slope} = 5 = \frac{5}{1}$$

Transform Standard Form to Slope=Intercept Form to help with graphing

EXAMPLE

$2x + 4y = 8$	
$4y = -2x + 8$	Subtract $2x$ from both sides.
$y = -\frac{2}{4}x + 2$	Divide both sides by 4.
$y = -\frac{1}{2}x + 2$	Simplify.

FORMS OF A LINEAR EQUATION

$2x = y - 3$	
Standard form	$2x - y = -3$
Slope-intercept form	$y = 2x + 3$
Point-slope form	$y - 5 = 2(x - 1)$

BE SURE TO WORK THROUGH ALL THE EXAMPLES IN ONLINE SCHOOL

OFFLINE WORK:

- Read pages 133–135 in the reference guide.
- Complete Problems 1–25 odd on page 136.
- Complete Problems 2–28 even on page 136 for extra practice (optional).

- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*

Lesson 2 – Point-Slope Form

People use tools for various jobs. A saw, screwdriver, and wrench are useful, but not for pounding a nail into a wall. In the same way, the slope-intercept form of a linear equation is useful at times. However, there are better ways to express a line, especially when only the most basic information is known. In that case, the point-slope form is a more useful tool, just as a hammer is a more useful tool for pounding in nails.

The different forms of equations are useful in performing many mathematical processes, such as solving word problems and estimating the slope of a graph of a line.

POINT-SLOPE FORM: an equation of a line that passes through the points (x, y), has a slope m, and is given by $y - y_1 = m(x - x_1)$

Point-Slope Form of a Linear Equation
The point-slope form a of linear equation is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is any point on the line.

Identify the slope and a point on the line. $y - 5 = 2(x - 4)$

$$y - \underset{y_1}{5} = \underset{m}{2}(x - \underset{x_1}{4})$$

$m = 2$ $(x_1, y_1) = (4, 5)$

Once you write an equation in point-slope form, you can use transformations to write the equation in slope-intercept form. How? Solve for y.

Converting from Point-Slope Form to Slope-Intercept Form
Write the equation $y - 2 = 3(x + 5)$ in slope-intercept form.

Use the distributive property.

$y - 2 = 3(x + 5)$ $y - 2 = 3x + 15$	Perform the distributive property to eliminate the parentheses.

Isolate y.

$y - 2 + 2 = 3x + 15 + 2$ $y = 3x + 17$	Add 2 to each side and simplify.

WORK THROUGH ALL EXAMPLES IN THE ONLINE SCHOOL

WATCH ALL VIDEOS IN ONLINE SCHOOL

OFFLINE WORK:

- Read pages 137–140 in the reference guide.
- Complete Problems 1–27 odd on pages 141–143.
- Complete Problems 2–28 even on pages 141–144 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 3 – SKIP (Optional Lesson)

Lesson 4 – Equations from Graphs

Crime fighters depend on collecting factual information and then using logic and scientific tools to solve crimes. You will follow a similar procedure when you collect facts about a given line, apply logic and the rules of mathematics to those facts, and transform the information you've collected into an equation of the line.

There are **three forms of an equation of a line**. They are point-slope form, slope-intercept form, and standard form. If you know the slope of a line and at least one point on the line, you can write an equation for the line. You can then use transformations to write that equation in any other form.

Point-Slope Form

$y - y_1 = m(x - x_1)$
where m is the slope and (x_1, y_1) is a point on the line.

Slope-Intercept Form

$y = mx + b$
where m is the slope and b is the y-intercept.

Standard Form

$Ax + By = C$
where A, B, and C are integers and A and B are both nonzero.

NOTE: In Standard Form, A should not be negative, A and B shouldn't both be zero, and A, B, and C should be integers (no fractions)

WATCH ALL VIDEOS and WORK THROUGH ALL EXAMPLES in the ONLINE SCHOOL

The "Standard Form" for writing down a Linear Equation is

$$Ax + By = C$$

A shouldn't be negative, **A** and **B** shouldn't both be zero, and **A**, **B** and **C** should be integers.

Example: Put this in Standard Form:

$$y = 3x + 2$$

Bring 3x to the left:

$$-3x + y = 2$$

Multiply all by -1:

$$3x - y = -2$$

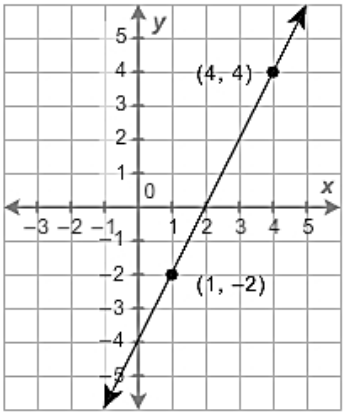
Note: A=3, B=-1, C=-2

There are three forms of the equation of a line: point-slope form, slope-intercept form, and standard form.

To write the equation of a line, you must know the slope and at least one point on the line.

When you are given two points on a line, you must first find the slope. If neither point is the y-intercept, begin by writing the equation in point-slope form. Then use transformations to write the other two forms of the equation.

Writing Equations of Lines from Graphs



$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{4 - 1} = \frac{4 + 2}{4 - 1} = \frac{6}{3} = 2$

Point-Slope Form: $y - y_1 = m(x - x_1)$

$y + 2 = 2(x - 1)$

Slope-Intercept Form: $y = mx + b$

$y = 2x - 4$

Standard Form: $Ax + By = C$

$-2x + y = -4$

OFFLINE WORK:

- Read pages 145–147 in the reference guide.
- Complete Problems 1–23 odd on pages 147–150.
- Complete Problems 2–24 even on pages 147–150 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 5 – CORE FOCUS – Sketching Lines

You can use a linear model to represent real-world situations that involve constant rates of change. When you know two points of a linear model or a point and the slope of a linear model, you can draw its graph. In this lesson, you will graph lines that model real-world situations given two points on the line or a point on the line and the line's slope. You will also practice sketching lines for situations for which you do not have actual values.

WATCH ALL VIDEOS and WORK THROUGH ALL EXAMPLES in the ONLINE SCHOOL

Graphing a Line from Two Points

Suzanne's car began with 25 gal of gas. After Suzanne drove 54 mi, the tank held 22 gal. Graph the amount of gas in the tank versus time.

Define variables:

Let m = miles driven
g = gallons of gas remaining

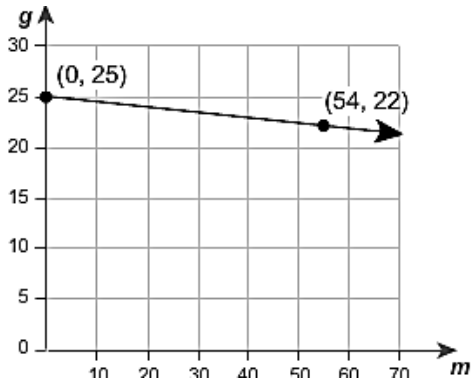
Since number of gallons of gas remaining, g, depends on number of miles driven, m, graph g along the vertical axis, and graph m along the horizontal axis.

Plot points:

$g = 25$ when $m = 0$, and $g = 22$ when $m = 54$. Plot the points (0, 25) and (54, 22).

Sketch a line:

Draw a line through the points (0, 25) and (54, 22). Since neither miles driven nor gallons of gas remaining can be negative, the line should only be shown in the first quadrant.



Graphing a Line Using a Point and the Slope

The water in Peter's pool was 49 in. deep. The depth decreased 0.8 in./day for 7 days. Draw a graph showing the depth of the water from the start of the week to the end of the week.

Define variables:

Let d = water's depth in inches
t = time in days

Since the water's depth, d, varies with the amount of time that passes, t, plot t along the horizontal axis and d along the vertical axis.

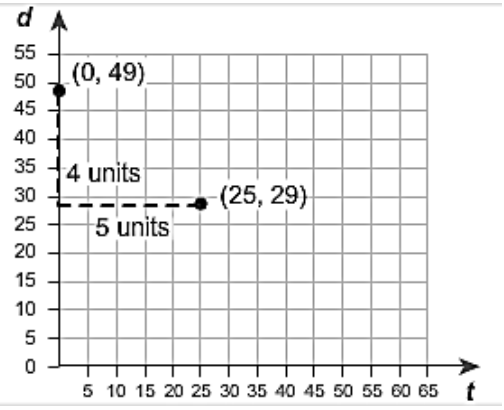
Plot the given point:

$d = 49$ when $t = 0$.
Plot the point (0, 49).

Plot another point:

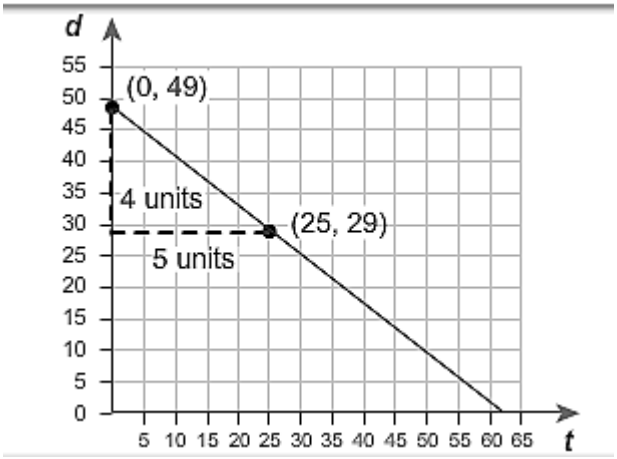
The rate of change in the pool's depth is −0.8. Write the slope as a fraction and use it to find the next point on the graph.

$$-0.8 = -\frac{8}{10} = -\frac{4}{5}$$



Sketch a line:

Draw a line through the points (0, 49) and (5, 45). Since neither depth of the water nor time in days can be negative, the line should only be shown in the first quadrant.



OFFLINE WORK

- Read pages 151–152.
- Complete Problems 1–3 on page 153.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 6 – SKIP – Optional Lesson

Lesson 7 – Applications: Linear Models

When you hear the word model*, you might think of something that represents or imitates something else. For instance, a model car is a smaller version of the original that it represents. In mathematics, a model is a mathematical representation of relationships that exist in a real-world situation. In this lesson, you will learn to model problems in which a linear relationship exists between the variables involved, and you will learn to identify when a model is linear.*

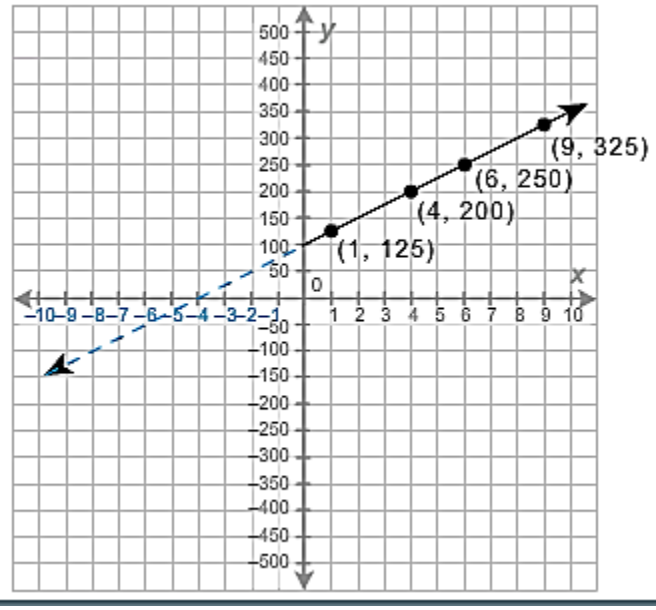
Many real-world problems can be solved by using equations. Sometimes the equations involve one variable, and sometimes they involve two. When an equation involves two variables, and the equation can be written in slope-intercept form, the equation and its corresponding graph together form a linear model of the problem you are trying to solve.

Linear Models
Balance

Francesca makes an initial deposit of $100 in her checking account. She then deposits $25 each month. Assuming she never makes a withdrawal, the balance in Francesca's account is given by the equation $B = 25t + 100$, where t is the time in months. Make a table of values for B when $t = 1$, 4, 6, and 9. Then graph the equation. Explain why this model is linear.

t	1	4	6	9
B	$B = 25t + 100$ $= 25 \cdot 1 + 100$ $= 25 + 100$ $= 125$	$B = 25t + 100$ $= 25 \cdot 4 + 100$ $= 100 + 100$ $= 200$	$B = 25t + 100$ $= 25 \cdot 6 + 100$ $= 150 + 100$ $= 250$	$B = 25t + 100$ $= 25 \cdot 9 + 100$ $= 225 + 100$ $= 325$

(1, 125)
(4, 200)
(6, 250)
(9, 325)



$B = 25t + 100$
$y = 25x + 100$

HOW DO YOU KNOW IT IS A LINEAR LINE?

- Look at the graph
- Look at the equation
- Look at the rate of change

Nonlinear Model
Ball Drop

The height of a ball dropped from a height h after t seconds is given by the equation $h = -16t^2 + 400$. Is the model linear?

t	1	2	4	5
h	$h = -16t^2 + 400$ $= -16 \cdot 1^2 + 400$ $= -16 \cdot 1 + 400$ $= -16 + 400$ $= 384$	$h = -16t^2 + 400$ $= -16 \cdot 2^2 + 400$ $= -16 \cdot 4 + 400$ $= -64 + 400$ $= 336$	$h = -16t^2 + 400$ $= -16 \cdot 4^2 + 400$ $= -16 \cdot 16 + 400$ $= -256 + 400$ $= 144$	$h = -16t^2 + 400$ $= -16 \cdot 5^2 + 400$ $= -16 \cdot 25 + 400$ $= -400 + 400$ $= 0$

(1, 384)
(2, 336)
(4, 144)
(5, 0)

rate of change $= \frac{y_2 - y_1}{x_2 - x_1}$

(1, 384)
(2, 336)
(4, 144)
(5, 0)

(1, 384) and (2, 336)

rate of change $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{336 - 384}{2 - 1} = \frac{-48}{1} = -48$

(2, 336) and (4, 144)

rate of change $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{144 - 336}{4 - 2} = \frac{-192}{2} = -81$

$-48 \neq -81$

NOT LINEAR

WATCH ALL VIDEOS and WORK THROUGH ALL EXAMPLES in the ONLINE SCHOOL

OFFLINE WORK

- Read pages 154–155.
- Complete Problems 1–27 odd on pages 155–157.
- Complete Problems 2–26 even on pages 155–157 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 8: CORE FOCUS – Linear Models

You have learned to write and graph linear equations using slope and points on a line. Situations in life can often be modeled using linear equations and their graphs. For instance, the graph of your growth over time would be a line. You could write an equation from the graph that models your growth.

In this lesson, you will write equations for real-world situations from word problems, tables, and graphs

WATCH ALL VIDEOS and WORK THROUGH ALL EXAMPLES in the ONLINE SCHOOL

Linear Equations from Word Problems
Using a Table

Bonnie purchased a bag of cat food. She created a table to show how much food remained after she fed her cat each day.

Days	1	3	7	11
Cups of food remaining	48	42	30	18

The rate of change, or slope, is −3.

Write an equation that represents the situation.

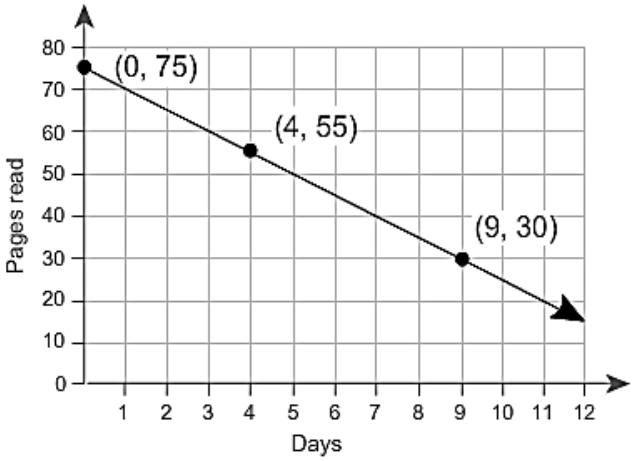
point-slope form: $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

m: −3
(x_1, y_1): (1, 48)

Substitute the values into point-slope form.

The equation is $y - 48 = -3(x - 1)$.

David has read 75 pages in his book and plans to read 15 pages per day. He drew a graph to show how many days it will take him to finish reading the book if he read at a constant rate.



Write an equation that represents the time David will spend reading his book.

Slope-intercept form: $y = mx + b$, where m is the slope and b is the y-intercept.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30 - 55}{9 - 4} = \frac{-25}{5} = -5$$

The equation is $y = -5x + 75$.

OFFLINE WORK:

- Read pages 158–159.

Complete Problems 1–3 on page 160.

- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*

Lesson 9: CORE FOCUS: Interpreting Linear Models

Real-world problems can often be modeled by linear equations. For instance, calculating your earnings at an hourly rate is a linear relationship. If you write an equation that represents how much you earn, you can use that equation to predict how much you will earn in the future or how much you can save over time. In this lesson, you will learn how linear equations can help you solve real-world problems.

WATCH ALL VIDEOS and WORK THROUGH ALL EXAMPLES in the ONLINE SCHOOL

OFFLINE WORK

- Read pages 161–162.
- Complete Problems 1–3 on page 163.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.