IMC – Unit 5: Systems of Equations

BE SURE TO WATCH ALL VIDEOS and WORK THROUGH ALL EXAMPLES within the OLS lessons

When two people meet, they often shake hands or say "hello" to each other. Once they start talking to each other, they can find out what they have in common. What happens when two lines meet? Do they say anything? Probably not, but whenever two lines meet, you know they have at least one point in common. Finding the point at which they meet can help you solve problems in the real world.

Lesson 1: Systems of Linear Equations

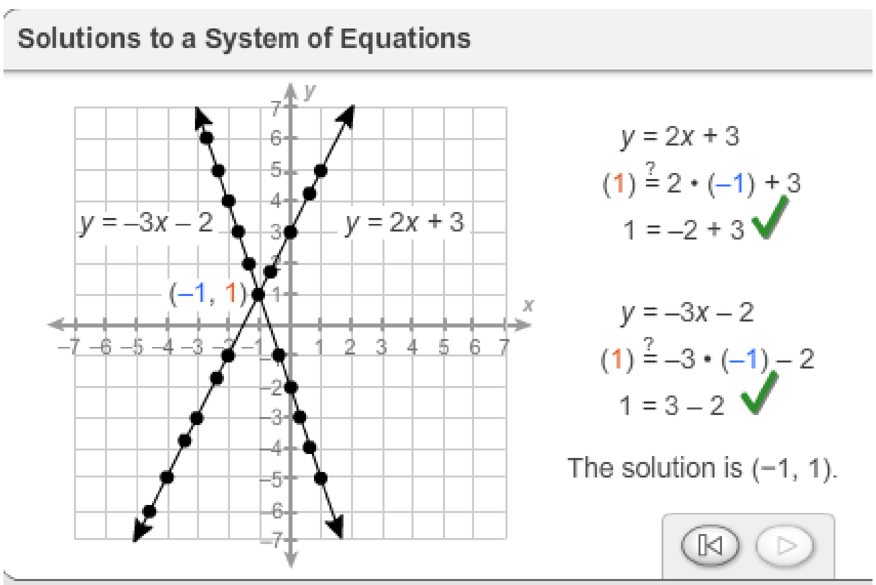
Suppose movie tickets for adults cost $9 each, and tickets for children cost $5. A movie theater sells 284 tickets for a show for a total of $2164. How many adult tickets and how many children's tickets did the theater sell?

You can use two equations with two variables to solve this kind of problem. In this lesson, you will learn how to identify solutions to systems of linear equations.

Vocabulary

A system of linear equations is two or more linear equations that use the same variables.

A solution of a system of linear equations is an ordered pair that makes all of the equations in the system true.



OFFLINE WORK

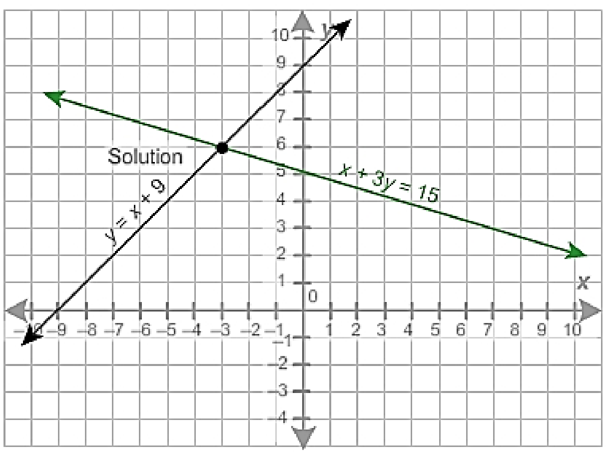
- Read pages 171–172 in the reference guide.
- Complete Problems 1–10 on pages 172–173.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 2: Solving Systems Using Inspection

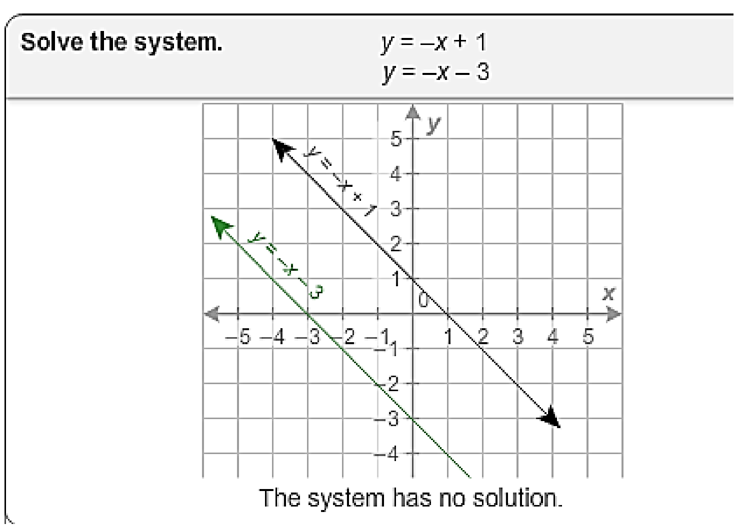
Solutions to systems of linear equations are ordered pairs (*x, y*). Graphically, a solution to a linear system is a point where the graphs of the equations intersect. So how many solutions do linear systems of equations have? Are you tempted to say that they have only one solution? That answer is true some of the time, but not all of the time. In this lesson, you will learn how to determine the number of solutions there are to a system of linear equations, and you will learn how to classify systems.

Vocabulary

A set of two or more equations with the same variables is a system of equations. If the graphs of all the equations are lines, it is a system of linear equations. An ordered pair is a solution of a system if it makes all the equations in the system true.



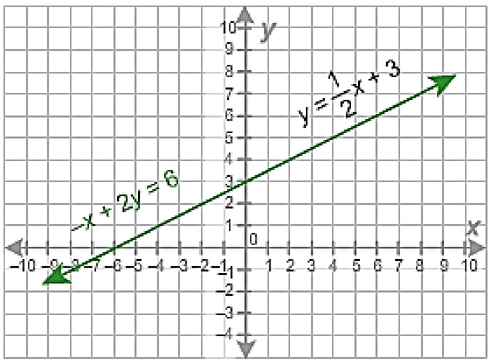
If the graphs of the equations in a system do not intersect, then the system does not have a solution. Although each equation has an infinite number of solutions, the system itself has no solutions.



Here, the graph of the second equation lies entirely on top of the graph of the first equation. These are equivalent equations. Notice that if you solve the second equation for y, you get the first equation.

Every point on the line makes both equations true, so the system has an infinite number of solutions

Solve the system. $y = 0.5x + 3$
$-x + 2y = 6$

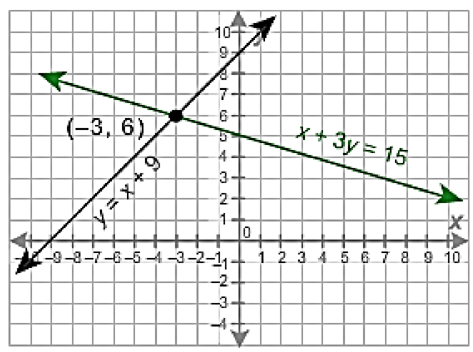


The system has infinitely many solutions.

You can classify a system of equations by the nature of its solution set.

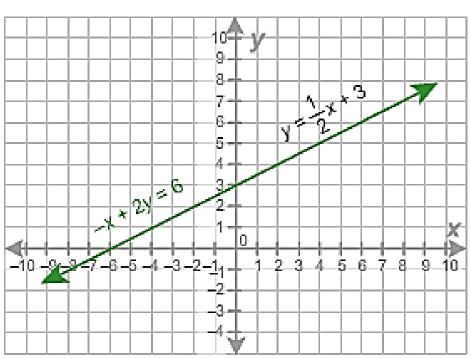
- If a system has at least one solution, then it is consistent.
  - If there is exactly one solution, the system is independent.
  - If there are an infinite number of solutions, the system is coincident. You can also call it dependent.
- If a system has no solution, then it is inconsistent.

Classifications
This system is consistent and independent.

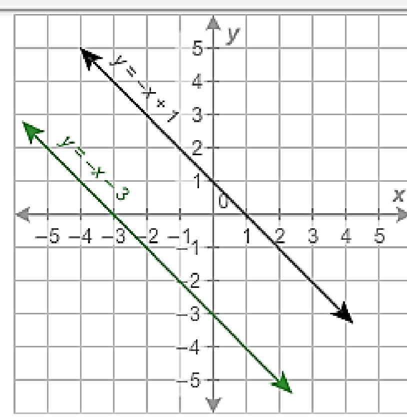


1 of 3

Classifications
This system is coincident.



Classifications
This system is inconsistent.



Classifying Systems by Inspection

Remember to first write both equations in the same form—either standard form or slope-intercept form.

Once the equations are in the same form, look for these features to identify the type of system and the number of solutions.

Inspecting Systems of Linear Equations

Inconsistent System
(no solutions)

Standard Form	Slope-Intercept Form
$2x + y = 3$ $2x + y = -5$ Sides of equations with variables are equal, but sides with constants are not.	$y = -2x + 3$ $y = -2x - 5$ The equations have the same slope but different *y*-intercepts.

Consistent Independent System
(one solution)

Standard Form	Slope-Intercept Form
$x + y = 7$ $3x + 2y = 7$ Sides of equations with variables are not equal, and neither equation is a multiple of the other.	$y = -x + 7$ $y = -\frac{3}{2}x + \frac{7}{2}$ The equations have different slopes.

Coincident System (Dependent)
(infinite solutions)

Standard Form	Slope-Intercept Form
$x + y = 2$ $2x + 2y = 4$ One equation is a multiple of the other equation or the two equations are identical.	$y = -x + 2$ $y = -x + 2$ The equations have the same slope and the same *y*-intercept.

OFFLINE WORK

- Read pages 174–176.
- Complete Problems 1–12 on pages 176–177.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 3: Using Graphs to Solve Systems

Solutions to a linear equation in two variables are the ordered pairs of the points that graph the equation's line. Solutions to a system of linear equations are the ordered pairs of points that the equations share. In this lesson, you will learn how to use the graphing method to solve systems of equations.

To solve a system of equations by graphing, graph each equation in the system, and look for the point of intersection. If the point of intersection does not have integer coordinates, you can estimate with the nearest point whose x- and y-coordinates are integers.

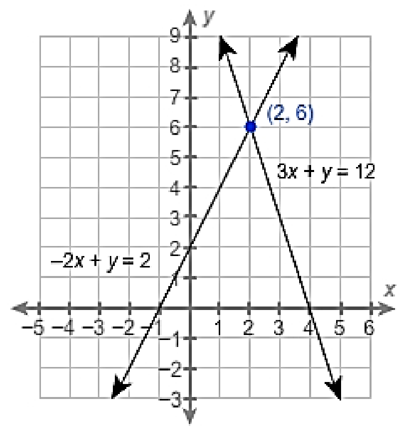
Using Graphs to Solve Systems

What is the solution to the system?

$-2x + y = 2$

$3x + y = 12$

The solution is (2, 6).



Solutions to systems of equations do not always have integer x- and y-coordinates. To estimate the solution, graph both equations and locate the point of intersection. Then, find the point with integer x- and y-coordinates that is nearest to the point of intersection.

VIDEO: MathCast: Solving a System
http://k12.http.internapcdn.net/k12_vitalstream_com/CURRICULUM/1277425/CURRENT_RELEASE/HS_A1G2_S1_08_01_WHB_SolveASystem.htm

OFFLINE WORK

- Read pages 178–179.
- Complete Problems 1–17 odd on pages 180–181.
- Complete Problems 2–18 even on pages 180–181 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 4: SKIP (Optional, Your Choice)

Lesson 5: Substitution Method

Need sugar? Some people like it in their food but want to remain healthy, so they use one of the many sugar substitutes that are available. In the same way, suppose someone asks you to represent the number 6 in six different ways. You could probably come up with more than just six ways in almost no time:

(4 + 2), $\frac{60}{10}$*,* $\sqrt{36}$*, a half-dozen, the whole number between 5 and 7,* $\frac{4 \cdot 12}{8}$ *, and so on.*

If you wanted to refer to the number 6, you could use any one of these substitutes, because they all equal 6. Substitution is the basis of another method for solving systems of linear equations.

Vocabulary

The substitution method is an algebraic method of solving a system of equations. It always identifies the solution with accuracy. Go to the next screen to see how it works.

The substitution method is an algebraic method of solving a system of equations. It is more reliable, and often quicker, than the graphing method.

To use the substitution method, isolate a variable in either equation, substitute its equivalent expression into the other equation, and solve for the other variable. Then substitute that value into either equation and solve for the remaining variable.

Solve the system.

$3x + 2y = 2 \qquad y = x - 9$

$3x + 2(x - 9) = 2 \qquad y = 4 - 9$

$3x + 2x - 18 = 2 \qquad y = -5$

$5x - 18 = 2$

$5x = 20$

$x = 4$

The solution is (4, –5).

VIDEO: Substitution Method
http://k12.http.internapcdn.net/k12_vitalstream_com/CURRICULUM/319952/CURRENT_RELEASE/MS_ALG_S1_08_02_WHB_substitution_method.htm

OFFLINE WORK

- Read pages 182–185.
- Complete Problems 1–27 odd on pages 185–186.
- Complete Problems 2–26 even on pages 185–186 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 6: CORE FOCUS: Solving a System from Points

You've seen linear systems expressed as two or more equations and as graphs of two or more lines. Linear systems can also be expressed as two or more sets of points. In this lesson, you will learn how to determine the number of solutions to a linear system from two sets of points

VIDEO: Solving a System from Points (within the OLS lesson)

Remember

- If the slopes are different, the lines intersect at one point. The system has one solution.
- If the slopes are the same but the y-intercepts are different, the lines are parallel and never intersect. The system has no solution.
- If the slopes are the same and the y-intercepts are the same, the lines are the same. The system has an infinite number of solutions.

OFFLINE WORK

- Read pages 187–189.
- Complete Problems 1–6 on page 189.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 7: SKIP (Optional , Your Choice)

Lesson 8: APPLICATIONS: Systems of Linear Equations

The world is full of problems that involve two variables—investments, motion, manufacturing, consumer purchases, and income, to name only a few. All can call for systems of equations in two variables. You have already practiced some strategies for solving such problems. In this lesson, you will apply your knowledge of systems of equations to real-world problem solving.

Plan for Solving a Word Problem	
Step 1:	Read the problem carefully. Decide what unknown numbers are asked for and what facts are known. Making a sketch (or using one of the other strategies you have learned) may help.
Step 2:	Choose variables and use them with the given facts to represent the unknowns described in the problem.
Step 3:	Reread the problem and write an equation that represents relationships among the numbers in the problem.
Step 4:	Solve the equation and find the unknowns asked for.
Step 5:	Check your results with the words in the problem. Give the answer.

If a real-world situation involves two unknowns and you have two facts, such as their total cost and their total value, or their total length and how they are related, then you can write a system of equations to solve for the unknowns.

Real-World Applications

Fishermen caught a total of 72 trout and salmon. They caught 3 times as many trout as salmon

$$t + s = 72$$
$$t = 3s$$

Real-World Applications

Lenny has 27 nickels and dimes worth a total of $1.75.

$$n + d = 27$$
$$0.05n + 0.1d = 1.75$$

<u>OFFLINE WORK</u>

- Read pages 190–192.
- Complete Problems 1–15 odd on pages 192–193.
- Complete Problems 2–14 even and 16–17 on pages 192–193 for extra practice (optional).
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 9: CORE FOCUS: Applications of Linear Systems

Suppose you're comparing two cell phone plans. For one plan, you'll pay $30 per month plus $0.10 per minute of use. For another plan, you'll pay $25 per month plus $0.15 per minute of use. After how many months will you pay an equal amount with both plans? This type of problem can be modeled using a system of linear equations. In this lesson, you will learn to solve these types of problems

VIDEO: Applications of Linear Systems (within the OLS lesson)

Work through this example in OLS

Applications of Linear Systems

To rent movies from an online rental site, people can pay $5 per rental or they can pay a one-time membership fee of $39 and rent movies for $2 per rental.

How many movies must you rent to pay the same amount on either plan?

A. Let c represent the total cost of renting movies, and let r be the number of rentals. Write an equation that models the cost of renting movies without a membership.

$c = 5r$

B. Write an equation that models the cost of renting movies with a membership.

$c = 39 + 2r$

OFFLINE WORK:

- Read pages 194–195.
- Complete Problems 1–3 on page 196.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 10: CORE FOCUS: Mixture Problems

Suppose you have a container of pure apple juice and a container of a cranberry and apple juice mixture that has 40% cranberry juice and 60% apple juice. You want to combine the pure apple juice and the cranberry-apple juice mixture to create 4 cups of a new mixture that contains 80% apple juice. How much of each type of juice would you need to use? This type of problem is a mixture problem. Mixture problems involve combining products that contain percentages of a particular ingredient. You use systems of linear equations to solve them. In this lesson, you will learn how to solve mixture problems.

VIDEO: Mixture Problems (within the OLS lesson)

OFFLINE WORK:

- Read pages 197–199.
- Complete Problems 1–3 on page 199.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 13: EXTENDED PROBLEMS: Reasoning

In this lesson, you'll complete Extended Problems: Reasoning for the Systems of Equations unit.