MATH C – Unit 8 – LINEAR MODELS

Lesson 1 – ***Semester Introduction:***

If I save $5.00 a week, how much will I have in a year? What if I save $10.00? What if I save for 2 years?

You can use mathematics to answer questions like these about your savings. When you use mathematics to describe relationships in the real world, you are creating a mathematical model.

As you will learn in this unit, you can model many relationships in the real world with simple lines.

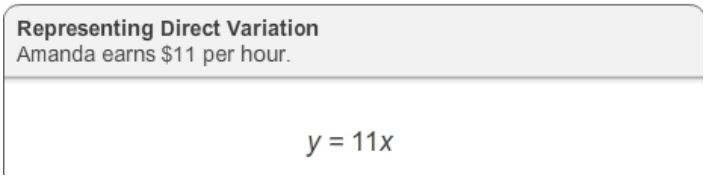
Lesson 2 – ***Direct Linear Variation, Part 1:*** *When you buy gas for a car, the amount you pay is based on the number of gallons you buy. If you buy twice as many gallons as your friend does, then you will pay twice as much as your friend does. If you buy three times as many gallons, then you will pay three times as much, and so on. This type of relationship is called a direct linear variation and is a special type of function.*

Objectives:

- Solve a problem involving direct linear variation.
- Find the constant of variation when given data for a function with direct linear variation.
- Write a formula for a problem involving a direct variation.

If a relationship is a direct linear variation, then every output is found by multiplying the input by a constant amount. The constant amount is called the constant of variation. It can be any number except zero.

When there is a direct variation between two quantities, you say that the output *varies directly* with the input. For example, if Amanda earns $11 per hour, you would say that her earnings vary directly with the number of hours she works.



** WATCH: Direct Variation in OLS

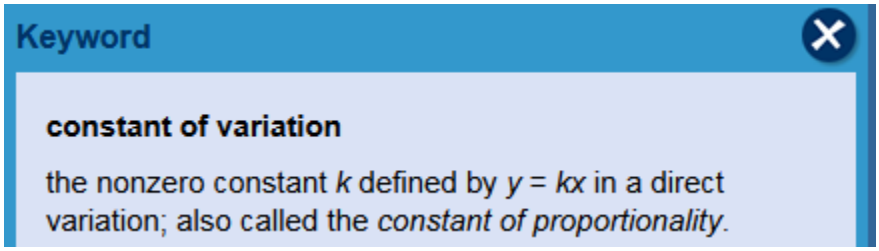
OFFLINE WORK

- Read pages 253–255.
- Complete Problems 1–21 odd on pages 255–256.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets.*

Lesson 3 – ***Direct Linear Variation, Part 2***: Problems involving direct variation cover a wide range of fields, including construction, landscaping, architecture, manufacturing, business, banking, electronics, computers, and science. In this lesson, you will apply what you already know about direct variation to solving practical problems

Objectives

- Solve a problem involving direct linear variation.



A direct linear variation is described by the equation

$y = kx$,

where k is the constant of variation.

** WATCH - MATH CAST: Paychecks in OLS

OFFLINE WORK

- Read page 255.
- Complete Problems 23–28 on page 256.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 4 – ***Quadratic Variation*** *- You've just learned about direct linear variation; however, not every relationship is linear. In this lesson, you are going to learn about another type of relationship called quadratic variation. As in the lessons on direct linear variation, you are going to learn the general equation for a quadratic variation. You will learn how to determine whether or not a relationship shows a quadratic variation, and you will learn how to graph a quadratic variation.*

Objectives

- Find the constant of variation when given data for a function with quadratic variation.
- Solve a problem involving quadratic variation.
- Write a formula for a problem involving a quadratic variation.
- Graph a function involving quadratic variation.

In this lesson, you will learn about a relationship between variables called **quadratic variation.** You will find that the techniques used in this lesson are very similar to those used in the lessons on direct linear variation.

EXAMPLE

Jan worked 7 hours last week and earned $28. Her wages vary directly in relation to the number of hours she works. Find a formula for the variation.

Set up the variables and the equation.

Let a equal the amount Jan earns.
Let h equal the number of hours she works.

$\therefore a = kh$

Substitute the given values and solve for k.

$28 = 7k$
$4 = k$

The formula is $a = 4h$.

Here's one more example of direct variation.

A Direct Variation Example
Given that m varies directly with n and that $m = 52$ when $n = 2$, find the constant of variation k and the value of m when $n = 4$.

$m = kn$ — Since there is a direct variation, you will write that $m = kn$.

$52 = k \cdot 2$ — Substitute an ordered pair you know, then solve for k.
$52 = 2k$
$26 = k$

$m = kn$ — Now, use the constant of variation to find m.
$m = (26)(4)$
$m = 104$

The constant of variation k is 26, and the value of m when $n = 4$ is 104.

QUADRATIC VARIATION: A relationship between x and y is a quadratic variation if you can write the function describing the relationship in the form of a general equation.

$$y = kx^2, \text{ where } k \text{ is a nonzero constant.}$$

How can you determine whether a relation is a quadratic variation?

If the relation is given as an equation, solve for y to determine whether it has the general form $y = kx^2$.

If the relation is given as a set of x- and y-values, simply divide each y-value by the square of the x-value to determine a value for k:

$$\frac{y}{x^2} = k$$

If you get the same value for k each time, the relation shows a quadratic variation.

OFFLINE WORK

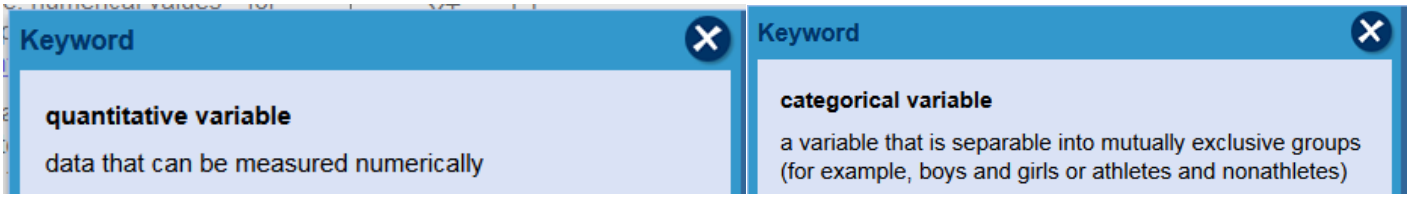
- Read pages 257–260.
- Complete Problems 2–16 even and 17–25 odd on pages 260–261.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets.*

Lesson 5 – OPTIONAL (Skip)

Lesson 6 – Patterns in Two-Way Tables: Data are collected and analyzed to solve problems and make decisions. One type of data is called categorical data. These data organize information in terms of groups or categories. In this lesson, you will learn to create tables that organize categorical data and to use those tables to analyze the data.

Objectives

- Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
- Use relative frequencies calculated for rows or columns to describe possible association between bivariate categorical data.



Types of Variables for a Car
Click to reveal each variable.
Think about what makes it either quantitative or categorical.

Quantitative	Categorical
age	color
cost	condition
length	make
number of previous owners	model

Keyword	Keyword
bivariate data data that show the relationship between two variables; paired data	**two-way table** a frequency table for two categorical variables

You can use a two-way table to calculate relative frequencies—the ratio of the frequency of a data value to the total number of data values in a given category.

For example, if 4 out of 5 people surveyed prefer low-fat milk to whole milk, the relative frequency of people who prefer low-fat milk is 4/5. Relative frequency can also be given as a percent: 80% of the people surveyed prefer low-fat milk.

Two-way relative frequency tables can help you determine whether there is an association, or relationship, between different categorical variables. You may want to convert each relative frequency to a percent in order to see the relationship more clearly.

OFFLINE WORK

- Read pages 262–265.
- Complete Problems 1–9 odd and 15–25 on pages 266–267.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets.*

Lesson 7 – Scatter Plots: *You may have noticed that the more active you are during the day, the more you sleep at night. Or you may have noticed that the more time you spend studying, the higher your test score is. Both situations describe bivariate data, or data that have two variables. In this lesson, you will learn to create scatter plots using bivariate data. You will use the plots to gather information about the data.*

Objectives

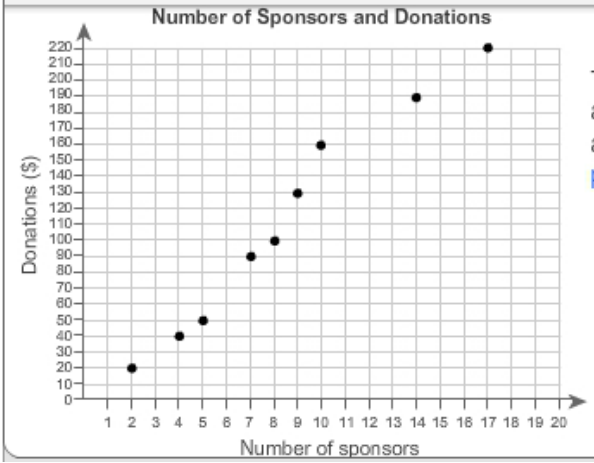
- Create scatter plots from given data.
- Interpret scatter plots.

A scatter plot is a graph that displays two-dimensional numerical data as points. Two-dimensional data consist of two variables.

You can write two-dimensional quantitative data as ordered pairs of numbers and then plot the ordered pairs on a coordinate grid.

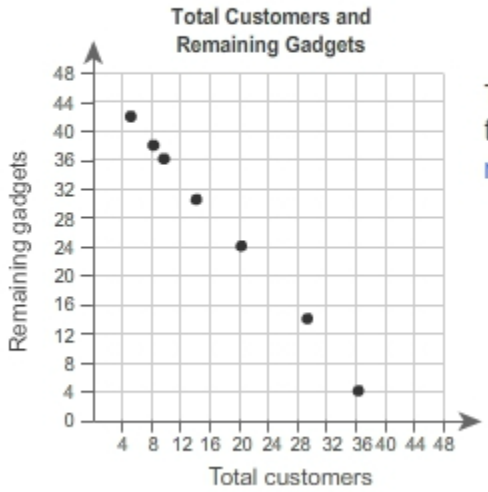
The data in scatter plots sometimes show a pattern. Look at these examples. Both graphs seem to follow a linear pattern. One has a positive slope, while the other has a negative slope.

Patterns in Data



The scatter plot appears to follow a line with a positive slope.

Patterns in Data



The scatter plot appears to follow a line with a negative slope.

OFFLINE WORK

- Read pages 268–271.
- Complete Problems 1–5 odd and 9–17 odd on pages 271–273.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets.*

Lesson 8 – OPTIONAL (Skip)

Lesson 9 – Clusters and Outliers

A clothing store sells different T-shirts at the following prices:

$15.24, $8.45, $7.98, $8.99, $7.89, $3.00.

You may notice that the prices $15.24 and $3.00 stand out compared to the other prices because they are, respectively, higher or lower. You may also notice that the other prices are clustered closer together. In this lesson, you will learn how to identify both clusters of data points and outlying points (points that lie apart from most of the other points) in data displayed in scatter plots.

Objective

- Identify clusters and outliers in a scatter plot

Scatter plots reveal information about the data they display. A plot may show a cluster, a group of points that are close together, or an outlier, a data value that is either much smaller or much larger than the other data.

To identify a cluster in a scatter plot:

- Look for data points that are grouped close together.
- Describe the cluster using the range of *x*- or *y*-coordinates that show the boundaries of its location.

To identify an outlier in a scatter plot:

- Look for any data points whose values are much larger or much smaller than those of the other data points.
- Describe the outlier using the variables that pinpoint its location.

OFFLINE WORK

- Read pages 274–275.
- Complete Problems 2–18 even on pages 276–277.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets*.

Lesson 10 – Associations in Scatter Plots: *When gathering evidence at a crime scene, investigators carefully look for footprints. A person's shoe size is related to his or her height, so investigators can measure a footprint to estimate a person's height. This information, along with the type of shoe and individual wear marks, is a clue that investigators can use to link a suspect to a crime.*

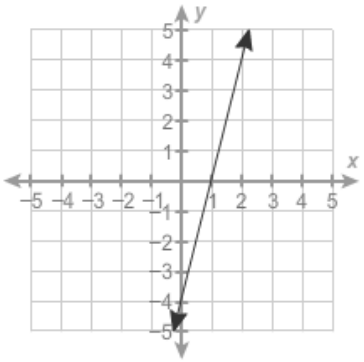
Many variables in our world have a relationship to, or an association with, each other. In this lesson, you will learn how to identify and describe associations by using scatter plots.

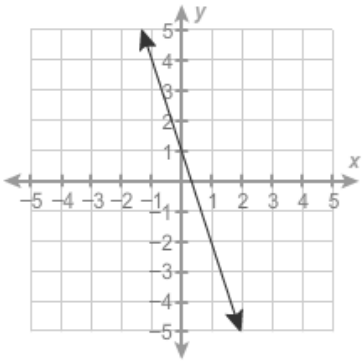
Objective

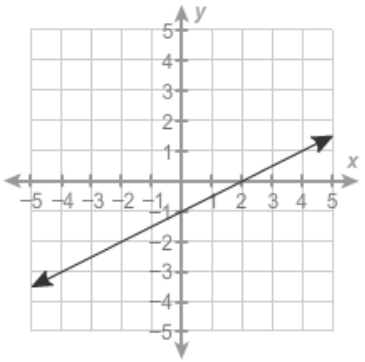
- Describe patterns in data

The slope of a line describes the direction of the line.

- A line with a positive slope moves upward from left to right. As one variable increases, the other variable also increases.
- A line with a negative slope moves downward from left to right. As one variable increases, the other variable decreases.

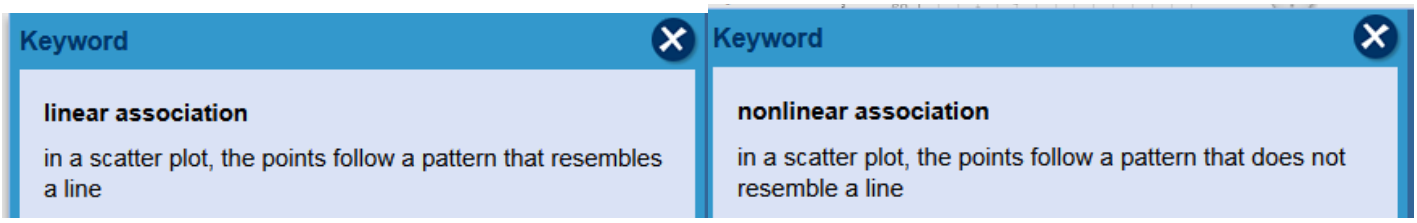






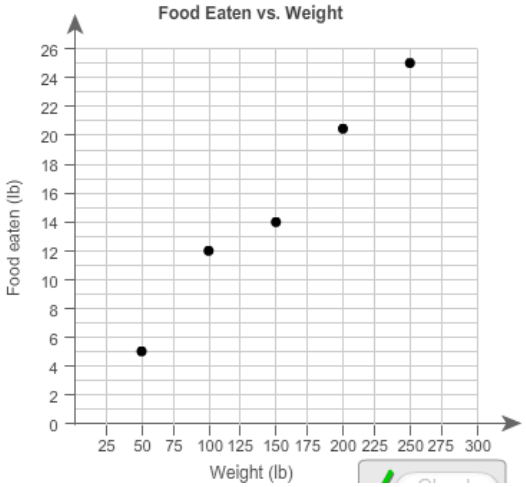
Scatter plots are widely used by researchers to determine whether two variables are related. A relationship between two variables is called an association. Two variables have

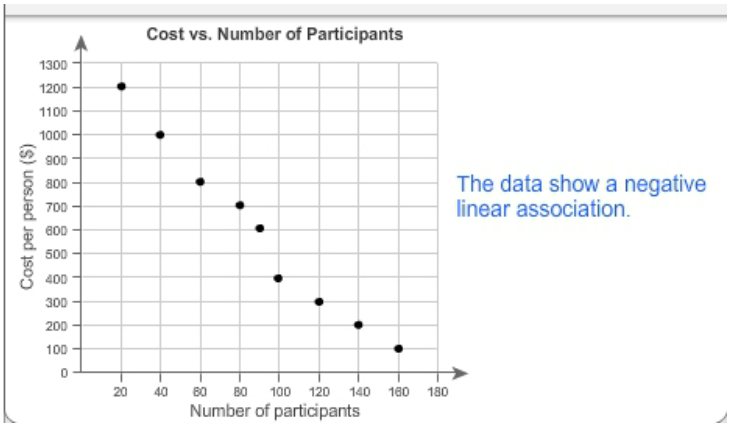
- a positive association when one variable increases as the other increases,
- a negative association when one variable decreases as the other increases, and
- no association when there is no pattern in the relationship.



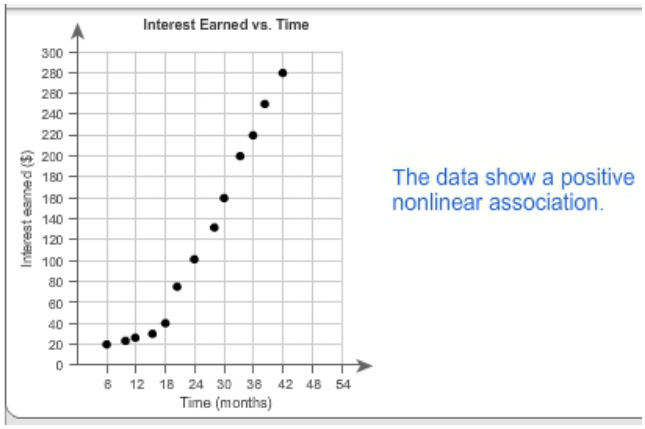
No Association

Positive Linear Association





The data show a negative linear association.



The data show a positive nonlinear association.

OFFLINE WORK

- Read pages 278–282.
- Complete Problems 1–19 odd on pages 282–284.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets.*

Lesson 11 – OPTIONAL (Skip)

Lesson 12 – Fitting a Line to Data: *Scientists might notice a negative linear pattern in a scatter plot that represents the population of a certain bird species. It would be helpful for them to be able to predict when the bird population will decrease to a certain point or even become extinct. Using a linear model of the data in the scatter plot, they can make such predictions. In this lesson, you will learn to draw an approximate linear model of the data in a scatter plot. You will also find the equation of your model and use that equation to make predictions about the data the line represents.*

Objective

- Use a linear model to approximate relationships between two quantitative variables.

LEARN: Drawing and Using Lines

Once you have the linear model, you can write the equation of the line using the same methods you use for any other line on a coordinate grid. To write the equation,

- Locate two points on the line.
- Use the two points to find the slope of the line.
- Use the slope and one of the points to write the equation of the line in point-slope form.

Finding the Equation of a Linear Model

A testing center allows 4 h for people to complete a test that has a maximum score of 1200.

The scatter plot shows the length of time different people took to complete the test and their scores. Find the equation of the linear model.

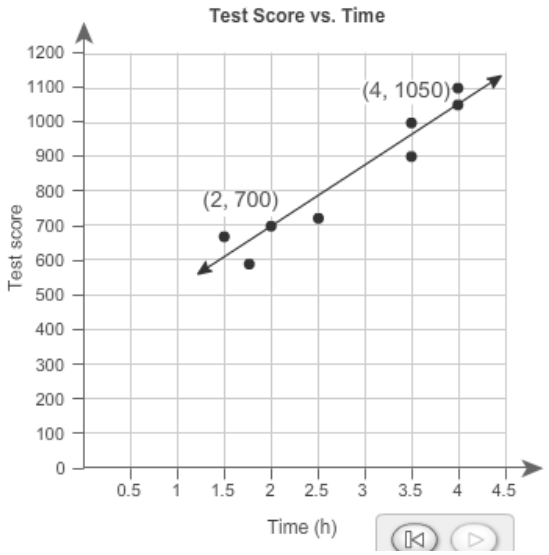
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1050 - 700}{4 - 2} = \frac{350}{2} = 175$$

$$y - y_1 = m(x - x_1)$$

$$y - 700 = 175(x - 2)$$

$$y - 700 = 175x - 350$$

$$y = 175x + 350$$



WATCH: MATH CAST: Linear Model in OLS

OFFLINE WORK

- Read pages 285–289.
- Complete Problems 2–20 even on pages 290–292.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets.*

Lesson 13 – CORE FOCUS: *Interpreting Slopes and Intercepts: When purchasing a new car, a buyer might research the cost of the car based on the features he or she wants. The buyer will discover that the cost of the car starts at a minimum price and increases as features are added. In math, the minimum price in this situation would be called the initial value, and the amount the price increases would be called the rate. In this lesson, you'll use linear models to interpret rate and initial value for different situations.*

Objective

- Interpret the slope as the rate for a linear model.
- Interpret the *y*-intercept as the initial value for a linear model.

WATCH: VIDEO LECTURE – Interpreting Slopes and Intercepts

OFFLINE WORK

- Read pages 293–294.
- Complete Problems 1–2 on page 295.
- Use the Solution Manual to check your work (optional). The Solution Manual is located in the Resources section in the Online Book Menu of *Intermediate Mathematics C: A Reference Guide and Problem Sets.*